

Proceedings of The Second International Conference on New Trends in Applied Mathematics, 2026

Beni Mellal, October 27-29, 2026

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Preface

It is with great pleasure that we present the proceedings of the **International Conference on New Trends in Applied Mathematics (ICNTAM 2026)**. This volume gathers a selection of the research presented at the conference and offers readers lasting access to recent results, innovative methodologies, and the emerging directions that are shaping the future of applied mathematics and scientific computing.

ICNTAM 2026 was conceived as a high-level scientific forum dedicated to the modern challenges of our discipline. Its program was built around the conviction that the most pressing mathematical questions of our time are rarely confined to a single subfield, and that progress increasingly depends on dialogue between theoretical insight and computational practice. Accordingly, the conference brought together internationally recognized mathematicians, researchers, and practitioners whose work spans mathematical modeling, inverse problems, partial differential equations, dynamical systems, numerical analysis, and the growing interface between mathematics and artificial intelligence. Through keynote lectures, technical sessions, and open discussion, participants engaged with both the formal structures that underpin these areas and the concrete problems—climate modeling, epidemiology, biology, image processing, data science, and complex systems—that motivate them.

A central aim of ICNTAM 2026 was to create a genuinely collaborative environment in which interdisciplinary research could take root. By assembling senior scientists, early-career researchers, and doctoral students on a single international platform, the conference sought to foster long-term collaborations, joint research initiatives, and the cross-fertilization of ideas across disciplinary boundaries. We are particularly gratified that the program offered young researchers and faculty members a setting for visibility, mentorship, and meaningful engagement with the global applied mathematics community—an investment in the next generation of scholars that we regard as one of the conference’s most valuable contributions.

The chapters collected in this volume reflect the breadth and vitality of that exchange. They range from foundational contributions in analysis and numerical methods to applied studies in which mathematical modeling meets data-driven and learning-based approaches. Each paper was evaluated for scientific quality and relevance, and we are grateful to the authors for entrusting their work to these proceedings and for the care they have taken in its presentation.

A conference of this scope depends on the generosity of many. We extend our sincere thanks to the keynote speakers, whose lectures framed the scientific conversation; to the members of the program and organizing committees, whose diligence shaped the program; to the reviewers, whose constructive criticism strengthened every contribution; and to the institutions and sponsors whose support made the event possible. Above all, we thank the participants, whose curiosity, rigor, and collegiality gave ICNTAM 2026 its character.

We hope that this volume will serve not only as a record of the conference but as a resource and an invitation—to further inquiry, to new collaborations, and to the continued cross-fertilization of ideas that the field so clearly demands.

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Oral Session 1. Machine Learning,
Data Science and Mathematical
Education

Residual Sinkhorn Transport Features for Kernel-Free Support Vector Classification

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Abstract:

Optimal transport provides a natural geometry for structured nonnegative data by comparing observations through displacement of mass rather than coordinate-wise similarity. However, repeated multidimensional transport computations may be costly, while nonlinear support vector machines usually rely on pairwise kernel evaluations whose prediction complexity can depend on the training set. We propose a Residual Sinkhorn Transport Support Vector Machine (RST-SVM), an explicit kernel-free classifier that combines finite entropically regularized optimal-transport coordinates with a weak aligned residual representation. A common reference measure is transported toward each observation by a finite number of Sinkhorn scaling iterations, and the resulting barycentric displacement is vectorized to form an explicit transport feature. This block is standardized and concatenated with a low-weight residual descriptor before training a single multiclass linear SVM. The construction separates the nonlinear geometric transformation from the final maximum-margin decision rule and avoids forming a nonlinear Gram matrix. For the finite Sinkhorn representation, we obtain an explicit norm bound that accounts for the row-marginal truncation error, together with a deterministic feature-space margin stability certificate. On a balanced handwritten-digit benchmark evaluated by leakage-free nested cross-validation, RST-SVM achieves $97.88\% \pm 0.64\%$ accuracy, compared with $97.76\% \pm 0.79\%$ for Sinkhorn-only features and $96.18\% \pm 0.72\%$ for a residual-only linear SVM. Its accuracy is close to that of an RBF-SVM while preserving an explicit finite-dimensional representation and showing improved robustness under additive noise and spatial perturbations. These results indicate that finite entropic transport features offer a practical bridge between optimal-transport geometry and scalable maximum-margin classification.

Keywords: optimal transport, Sinkhorn algorithm, support vector machine, entropic regularization, explicit features, image classification

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Automated Selection of Synthetic Data Generators for Medical Data

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Abstract:

The growing availability of medical data offers significant opportunities for healthcare applications, while its exploitation remains limited by privacy concerns, data heterogeneity, and restricted accessibility. Synthetic data generation has emerged as a promising solution for producing artificial data that preserve relevant characteristics of real data. However, the effectiveness of generative models strongly depends on their configuration and on the characteristics of the considered data, making the selection of an appropriate model challenging. This work proposes an Automated Machine Learning (AutoML)-based approach to automate the selection and optimization of synthetic data generators for medical data. The approach combines automated hyperparameter optimization with statistical and predictive evaluation to identify suitable generative configurations. Experimental results demonstrate the effectiveness of the proposed approach in identifying appropriate generators and producing synthetic data with useful statistical and predictive properties. This work provides an automated framework for supporting synthetic data generation in healthcare-oriented machine learning applications.

Keywords: AutoML, Synthetic Data, Medical Data, Generative Models, Hyperparameter Optimization

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Prediction Reliability in Cross-System Log Anomaly Detection

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Abstract:

Log anomaly detection models have demonstrated strong performance on widely used benchmark datasets. However, these models are commonly trained and evaluated within the same system, whereas real-world deployment may require a detector trained on one software environment to operate on logs generated by a different and previously unseen system. Although recent cross-system log anomaly detection approaches mainly focus on improving predictive performance and domain transfer, less attention has been given to the reliability of model predictions under such distribution shifts. This study investigates whether the confidence produced by log anomaly detectors remains trustworthy when models are transferred across heterogeneous logging environments without target-domain retraining. In particular, we examine whether a detector can remain highly confident even when its anomaly detection performance deteriorates on an unseen system. Models are trained on a source log dataset and evaluated both within the original environment and across different log systems, including HDFS, BGL, and Thunderbird. The study aims to analyze the relationship between predictive performance and confidence reliability under cross-system distribution shift and to determine whether conventional confidence estimates are sufficient for identifying unreliable predictions. The expected findings can provide a foundation for developing reliability-aware log anomaly detection approaches capable of recognizing when predictions on previously unseen systems should be trusted, rejected, or subjected to additional analysis.

Keywords: Log anomaly detection, Cross-system generalization, Prediction reliability, Distribution shift, Uncertainty estimation, System logs

Deep Clustering for Latent Structure Discovery in High-Dimensional Data

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Abstract:

The increasing dimensionality and complexity of modern datasets make traditional clustering challenging, particularly when relevant structures are hidden in nonlinear feature spaces. Deep clustering addresses this limitation by jointly learning compact representations and meaningful cluster structures in an unsupervised manner. This work investigates the use of deep representation learning for discovering latent structures in high-dimensional data. An autoencoder-based deep clustering framework is employed to simultaneously reduce dimensionality and organize observations into coherent groups in the learned latent space. The analysis focuses on the evolution of the latent representation during training and on how initially overlapping observations progressively form more compact and separated clusters. The learned representations are examined through dimensionality-reduction visualizations and internal clustering criteria, including cluster compactness, separation, silhouette coefficient, and reconstruction behavior. Particular attention is given to the interaction between representation learning and cluster refinement, highlighting how the latent space is progressively reorganized according to the underlying structure of the data. The experimental analysis illustrates the potential of deep clustering as an unsupervised exploratory tool for complex high-dimensional datasets. Beyond dimensionality reduction, the results show how deep representations can facilitate the identification and interpretation of hidden data structures without requiring predefined labels.

Numerical word puzzles as a prerequisite to mathematical thinking

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Abstract:

Numerical word puzzles offer an accessible and engaging way to initiate mathematics outreach activities. Rather than relying on conventional self-introductions, participants introduce themselves through short numerical clues that invite discussion, logical reasoning, and collaborative problem solving. This transforms a routine icebreaker into an opportunity for mathematical exploration before the workshop formally begins. By connecting simple numerical observations to broader mathematical ideas, such activities encourage curiosity, participation, and confidence among audiences with diverse mathematical backgrounds. Their informal nature lowers barriers to engagement while demonstrating that meaningful mathematical insights can emerge from elementary numerical reasoning. As an introductory activity, numerical word puzzles provide an effective means of broadening access to mathematical thinking and setting an improved mindset in the workshop that follows.

Keywords: Popular mathematics, Workshop, Numerical word puzzles, Epistemology

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Gated Fusion of Transformer Models for Suicidal Ideation Detection in Moroccan Darija YouTube Comments

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Abstract:

Detecting suicidal ideation in user-generated content written in under-resourced languages remains challenging, especially when the same language appears in different scripts and informal writing forms. In this work, we investigate the detection of suicidal ideation in Moroccan Darija YouTube comments through a three-class classification into Normal, At-Risk, and Suicidal categories. We develop a data-driven deep learning approach based on complementary transformer representations to handle the linguistic heterogeneity of Moroccan Darija. In this study, we collect and annotate 909 Moroccan Darija YouTube comments. The dataset contains Moroccan Darija written in Arabic script, Moroccan Darija written in Latin characters (Arabizi), and some comments that combine both forms. For the classification, XLM-R, MARBERTv2, and DarijaBERT-mix are first evaluated independently. We then investigate the same fusion approach using XLM-R combined separately with MARBERTv2 and DarijaBERT-mix. For each pair, we evaluate simple concatenation and a gated fusion mechanism, in which the representations produced by the two transformers are combined through a learnable gating function to capture their complementary information. The experimental results show that the highest-performing model is the gated fusion of XLM-R and MARBERTv2, achieving 88.24% accuracy, 85.25% balanced accuracy, 89.39% macro-precision, and 86.69% macro-F1 on the test set. Compared with the gated fusion of XLM-R and DarijaBERT-mix, which achieves 83.09% accuracy and 80.00% macro-F1, the proposed XLM-R and MARBERTv2 fusion improves accuracy by 5.15 percentage points and macro-F1 by 6.69 points. These results suggest that combining multilingual and Arabic-oriented transformer representations can improve the classification of heterogeneous Moroccan Darija content. The proposed approach provides a data-driven framework for suicidal ideation detection.

Keywords: Moroccan Darija, Suicidal Ideation Detection, Transformer Models, Gated Fusion, Deep Learning

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A Robust Twin Support Vector Machine with Coarse Representation and Complementary Feature Integration

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Abstract:

Support vector machines (SVMs) and their twin variants are widely used for binary classification, yet their performance deteriorates when training samples are contaminated by noise or outliers, and when the available feature representations are incomplete or heterogeneous. This work proposes a robust twin SVM framework that combines coarse data representation with complementary information drawn from multiple feature sources. The coarse representation groups nearby samples into compact regions, reducing the influence of individual noisy points and lowering the effective size of the training problem. Complementary information from multiple feature sources is then integrated to refine the decision boundaries and improve generalization. The resulting model retains the computational advantages of twin-type formulations while gaining robustness against noise and outliers. We outline the main modeling ideas, discuss the properties of the proposed formulation, and report experimental results on benchmark datasets under varying noise levels and feature configurations. The proposed framework is particularly relevant to real-world problems involving noisy data, such as medical diagnosis, fault detection, fraud detection, and imbalanced document classification. Future work will extend the framework to multi-class and large-scale settings.

Keywords: Twin support vector machine, Robust classification, Pattern recognition, Complementary feature integration, Coarse representation

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Bounded-Influence Distance-Weighted Discrimination: Geometric Reweighting for Robust Large-Margin Classification

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Abstract:

Distance-Weighted Discrimination (DWD) was developed for the High-Dimensional Low Sample Size (HDLSS) regime, where letting every observation influence the separating hyper-plane avoids the data piling of Support Vector Machines. We show that this same property leaves DWD without robustness. Although the derivative V'_q of the generalised DWD loss is bounded, it does not redescend: on the misclassified branch $V'_q(m) = -1$ exactly, so the score retains the leverage factor $\|x\|$. We prove that a high-leverage misclassified observation therefore has influence growing linearly in $\|x\|$, and that no fixed class weight can bound it. We introduce the weighted loss $L(w, b) = \sum_i \alpha y_i \varphi(d_i) V_q(y_i(w^\top x_i + b)) + \lambda \mathcal{R}(w)$ and prove that a geometric weight $\varphi(d_i)$ decaying with a robustness statistic bounds the direct influence channel, with a redescending contaminant contribution under an additional tail condition on φ . The curvature operator is shown to be nonsingular on the tangent space of the sphere rather than assumed so, and for fixed weights the objective stays convex with a globally Lipschitz gradient of constant $(q+1)^{2/q}$; joint estimation of the weights remains an open algorithmic question. The analysis is conditional on the preliminary location-scatter functional used to build d_i . On low- to moderate-dimensional benchmarks under targeted leverage contamination, the weighted estimator holds AUC near 0.994 from 0 to 15% contamination while standard DWD falls to 0.866, and estimating the influence function directly recovers the predicted rates: log-log slopes of +0.96 without the geometric weight and -1.07 with it. A dimensionality stress test shows that the particular regularised Mahalanobis statistic used here becomes nearly non-discriminative when $p \gg n$, which identifies a limitation of that construction rather than of the influence-control mechanism.

Keywords: Distance-weighted discrimination, Influence function, Bounded influence, Robust statistics, Redescending weights, Convex optimisation, Large-margin classification, HDLSS

Class-Labelled Generative Landscape Regularization for Incomplete Multi-View Classification under Imbalance and Label-Dependent Missingness

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Abstract:

Incomplete multi-view classification becomes particularly challenging when class imbalance and view availability are coupled: minority and frequent classes can be learned from systematically different subsets of observed views. Existing methods address missing-view recovery, uncertainty, or class imbalance from complementary perspectives. We study whether a shared class-labelled latent reference can improve a fixed imputation-based classifier without replacing its prediction mechanism. We introduce Landscape-Compact, a lightweight normalized Gaussian-mixture regularizer added to the shared latent space of a RIML-inspired host. One component is associated with each class. A supervised responsibility loss anchors latent components to labels, while a marginal latent-density loss regularizes the fused and view-specific representations. The mixture is used during training only; inference remains on the host's discriminative, imputation-based classifier. For an unweighted labelled code, responsibility cross-entropy plus marginal negative log-density reduces by Bayes' rule to the negative joint latent density, providing a direct link between the two regularization terms and class-centred latent geometry. Evaluation covers nine multi-view datasets. A matched benchmark with 102 protocol cells and five optimization seeds per cell yields mean macro-F1 advantages of 0.941 percentage points over the matched host, 6.758 over TCVC, and 10.936 over DCP. A controlled density-loss intervention shows that joint training outperforms either density term alone, while a zero-pair co-observation stress test consistently reduces class-centred cross-view gaps without a corresponding mean predictive gain. These results support Landscape-Compact as a reproducible, dataset-dependent latent regularizer rather than a universally superior classifier.

Keywords: incomplete multi-view classification, class imbalance, label-dependent missingness, latent regularization, Gaussian mixtures, information fusion

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ARASC: An Agentic Relationship-Aware Semantic Cleaning Framework for Data Cleaning with LLM

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Abstract:

Real-world tabular data often contains inconsistencies, and removing these for reliable data preparation is a major hurdle in pipelines. Despite LLMs facilitating innovative automated cleaning methods, current systems base their reasoning on Functional Dependencies (FD), overlooking patterns frequently encountered in real scenarios. We present ARASC (Agentic Relationship-Aware Semantic Cleaner), an LLM-based agentic framework that formalizes and exploits six inter-column relationship types — FD, Conditional Functional Dependencies (CFD), Extraction, Combination, Surrogation, and Calculation — through a three-agent pipeline: a semantic Profiler, a code-generating Developer, and a deterministic Validator. Evaluated against state-of-the-art baselines — Cocoon, IterClean, and GIDCL — ARASC achieves the highest F1 on Hospital (0.969 vs. 0.944) and Beers (0.997 vs. 0.983), and closely matches the best system on Flights (0.965 vs. 0.973), all without labeled training data, human annotation, or dataset-specific rule authoring.

Keywords: data cleaning, LLM, agentic framework, Functional Dependencies, Conditional Functional Dependencies, semantic cleaning

Clayton Copula Regularization for Neural Topic Models on Short Texts: Analysis, Experiments and Extension

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Abstract:

On short texts, sparse word co-occurrences lead neural topic models to infer diffuse topic distributions. We study CR-GSM, introduced by Lin et al., which formulates topic inference as a regularized variational optimization problem: the evidence lower bound of the Gaussian Softmax Model is maximized jointly with a penalty derived from the Clayton copula and applied to the topic proportions of each document, which encourages them to concentrate on a few topics. The model is trained with Adam and the reparameterization trick, for 10, 20 and 50 topics. We show that the shift constant of the penalty is not a mere rescaling: it defines an explicit concentration threshold beyond which the regularization switches off. Experiments on SearchSnippets and StackOverflow show that the regularization improves NPMI coherence by 37

Keywords: Regularized optimization, Variational inference, Clayton copula, Neural topic models, Short texts

Diabetes Prediction Using Machine Learning Algorithms

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Abstract:

Diabetes mellitus is one of the most serious chronic diseases affecting millions of people worldwide. Early diagnosis and prediction of diabetes are essential for reducing complications and improving patient quality of life. This study presents a framework based on modern machine learning (ML) and deep learning (DL) algorithms for diabetes prediction using the Association of Diabetics of Beni Mellal city (ADBMD) Dataset. The proposed framework evaluates Logistic Regression (LR), Random Forest (RF), Support Vector Machine (SVM), Extreme Gradient Boosting (XGBoost), Light Gradient Boosting Machine (LightGBM), Deep Neural Network (DNN), and Attention-Based Neural Network for Tabular Data (TabNet). Experimental results demonstrate that advanced boosting and attention-based models, particularly LightGBM and TabNet, achieve superior predictive performance compared to traditional approaches. The originality of this work lies in the unified comparative evaluation of classical ML, boosting methods, DL architectures, and TabNet under the same preprocessing and evaluation framework. In addition, the study emphasizes both predictive accuracy and interpretability for medical decision support systems.

Keywords: Diabetes Prediction, Deep Learning, XGBoost, LightGBM, TabNet, Support Vector Machine, Random Forest, ADBMD Dataset

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MOLG³-SAGE: A Novel Graph Neural Network Architecture for Mutagenicity Prediction Using Geometric Graph Scattering

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Abstract:

This paper tackles the critical challenge of mutagenicity prediction by introducing three approaches. First, we demonstrate that 2D scattering coefficients derived from molecular images outperform traditional molecular descriptors. Second, we propose a hybrid framework unifying geometric graph scattering (GGS), Graph Isomorphism Networks (GIN), and machine learning models, achieving strong mutagenicity prediction. Third, we design, MOLG³-SAGE, a novel graph neural network architecture which integrates GGS node features into a fully connected graph structure, delivering state-of-the-art predictive performance. Experimental results on the Hansen et al. dataset reveal that our methods surpass existing benchmarks, emphasizing the effectiveness of combining 2D and geometric scattering transform with graph neural networks. This study illustrates the potential of GNNs and GGS for mutagenicity prediction, with broad implications for drug discovery and chemical safety assessment.

Keywords: Mutagenicity, Graph neural networks, Geometric scattering transform, Toxicity

A Temporal Data-Driven Framework for Premier League Match Outcome Prediction Using Historical Performance and Market Probabilities

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Abstract:

Football match-outcome prediction is a challenging multiclass problem due to the temporal evolution of team performance and the uncertainty associated with home wins, draws, and away wins. This study proposes a leakage-resistant temporal machine learning framework for predicting English Premier League match outcomes using historical team-performance indicators and betting-market probabilities. Data from six consecutive seasons (2020-2021 to 2025-2026), comprising 2,280 matches, were analyzed. Historical features were constructed from up to the five preceding matches using a one-match temporal shift to prevent information leakage. Relative home-away performance indicators were combined with normalized book-maker probabilities. Logistic Regression, class-weighted Logistic Regression, Random Forest, and Gradient Boosting were evaluated using a strictly chronological training-validation-test design. On the 2024-2025 validation season, the betting-market baseline achieved the highest Accuracy (0.538) and lowest Log Loss (0.974). The combined Logistic Regression model achieved comparable performance with 0.536 Accuracy, 0.421 Macro F1-score, and 0.991 Log Loss. On the unseen 2025-2026 season, it achieved 0.464 Accuracy and 0.350 Macro F1-score, with draw prediction remaining the main challenge. The results demonstrate the strength of market-based information and provide a reproducible temporal framework for evaluating data-driven football prediction models under realistic forecasting conditions.

Keywords: Machine Learning, Temporal Modelling, Football Analytics, Multiclass Classification, Betting Market, Data-Driven Methods

Scalable Heart Disease Prediction on Hadoop and Spark Clusters Using Distributed Machine Learning Algorithms

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Abstract:

The rapid growth of healthcare data requires scalable computing frameworks capable of processing large volumes of medical information efficiently. Traditional machine learning approaches often face limitations when handling large-scale datasets due to computational and storage constraints. This study investigates the performance of distributed machine learning algorithms using Apache Spark and Hadoop Distributed File System (HDFS) for heart disease prediction. A distributed cluster composed of one master node and two worker nodes was deployed using Apache Hadoop 3.2.3 and Apache Spark 3.2.4. Experiments were conducted on synthetic large-scale versions of the Heart Failure Prediction dataset, ranging from 100,000 to 5 million records. Three machine learning algorithms were evaluated, namely Logistic Regression, Random Forest, and Gradient Boosted Trees. The models were assessed using Accuracy, Precision, Recall, F1-score, Area Under the ROC Curve (AUC), and training time. Experimental results show that Random Forest achieved the best predictive performance with an accuracy of up to 99.47% and an AUC of 0.9999. Logistic Regression exhibited the lowest computational cost but lower predictive performance, with an average accuracy of approximately 85.8%. Although Gradient Boosted Trees produced competitive predictive results, its training time increased significantly for large datasets. Furthermore, scalability experiments demonstrated that adding a second worker node reduced the Random Forest training time by 36.2%, achieving a speedup of 1.57 and an efficiency of 78.5%. The findings confirm the suitability of Apache Spark for scalable medical data analytics and highlight Random Forest as the most effective model in terms of accuracy and computational efficiency.

Keywords: Apache Spark, Hadoop, Big Data Analytics, Distributed Machine Learning, Heart Disease Prediction, Random Forest, Healthcare Informatics, Scalability

A Post-hoc Calibration and Class-Conditional Conformal Safety Layer for Imbalanced Breast-Cancer Triage

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Abstract:

Breast-cancer screening classifiers deployed in telemedicine are typically over-confident and provide no per-class coverage guarantee. Under the severe class imbalance of screening (cancer prevalence below 10%), a conformal predictor can satisfy its marginal coverage target while leaving the minority cancer class under-covered — the unsafe failure mode. We present a post-hoc, distribution-free safety layer that augments an existing screening classifier without retraining: (i) probability recalibration, (ii) class-conditional (Mondrian) split-conformal prediction that guarantees coverage within the cancer class, and (iii) a calibrated risk stratification for triage. The layer is rank-preserving, so it leaves the underlying classifier’s discrimination unchanged and is therefore complementary to any such model, including multimodal fusion systems. On a patient-level held-out test at real screening prevalence (8.16%; 11,669 images), the layer reduces the Brier score from 0.063 to 0.039 and the expected calibration error from 0.051 to 0.025 (non-overlapping 95% bootstrap intervals), and attains 90.6% ([88.0, 93.3]) empirical coverage on the cancer class at a 90% target, versus a marginal procedure that does not guarantee the minority class. The highest-criticality tier (11% of the volume) concentrates 73.7% of all cancers. We argue that calibration must be assessed at deployment prevalence, and that class-conditional coverage — not marginal coverage — is the relevant safety property for imbalanced screening.

Recurrent Neural Networks and Gradient Boosting for Stock Price Forecasting: A Comparative Study

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Abstract:

Stock-price forecasting remains difficult because financial markets are noisy, nonlinear, and constantly evolving. This study compares three machine-learning models for predicting nextday closing prices: Long Short-Term Memory (LSTM), Gated Recurrent Unit (GRU), and Extreme Gradient Boosting (XGBoost). LSTM and GRU learn from the temporal structure of price sequences, whereas XGBoost uses an ensemble of decision trees. The analysis covers Amazon, Apple, Google, and Microsoft, using 2766 daily closing prices per stock collected from Yahoo Finance between January 2015 and December 2025. The models use the previous 60 trading days to predict the next closing price. To avoid data leakage, the observations are kept in chronological order, and normalization is based only on the training data. Performance is assessed using a chronological 80/20 train-test split and five-fold expandingwindow cross-validation, with MAE, RMSE, MAPE, and R2 as evaluation metrics. The results show that GRU is the most accurate and consistent model across the four stocks. It achieves hold-out R2 values between 0.958 and 0.976 and MAPE values between 1.66% and 2.47%. LSTM also provides competitive forecasts, although its performance is less stable. XGBoost, however, produces higher errors and negative R2 values. This result is partly explained by the strong rise in stock prices during the test period: depending on the stock, 58% to 97% of the test prices exceed the highest price observed during training. Such conditions are particularly challenging for tree-based models, which struggle to predict beyond previously observed values. Overall, GRU provides the most reliable forecasts for the four stocks examined in this study.

Limit Theorems for General Recursive Regression Models Involving Weakly Dependent Functional Data

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Abstract:

Functional data, such as curves recorded in meteorology or medicine, are increasingly common, but classical kernel estimators must reprocess the whole sample at each new observation, which is costly for large or streaming data sets. We study a general class of recursive kernel estimators of Robbins–Monro type for regression functionals $E(L(Y)|X = x)$, where X takes values in a normed space. These estimators are updated online, without storing past data. Instead of restrictive mixing conditions, we extend the weak dependence notion of Doukhan and Louhichi to functional variables, a framework that includes functional Bernoulli shifts, linear processes and functional GARCH models. Under general conditions, we derive the asymptotic variance, establish asymptotic normality—the first such result for weakly dependent functional data—and obtain almost sure convergence rates, which are optimal under geometric decay of the dependence coefficients. Applications to the estimation of the regression function and of the conditional distribution function, including asymptotic confidence bands, are presented.

Keywords: Functional data, Weak dependence, Recursive kernel estimation, Nonparametric regression, Asymptotic normality, Almost sure convergence, Conditional distribution function

Oral Session 2. Computer Vision,
Medical Imaging and Pattern
Recognition

PDE-Guided Lightweight Deep Learning for Medical Image Restoration

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Abstract:

Medical image restoration is an essential step for improving image quality and facilitating subsequent computer-aided analysis. However, deep learning-based restoration methods often involve a trade-off between restoration quality, preservation of relevant image structures, and computational complexity. In parallel, partial differential equation (PDE)-based approaches provide a mathematically grounded framework for regularizing image restoration and controlling the evolution of image structures. This work investigates a hybrid framework that combines the representation capabilities of lightweight deep learning architectures with PDE-inspired regularization for medical image restoration. The main objective is to explore whether mathematical guidance can improve the restoration capability of a lightweight network while maintaining a limited computational cost. The proposed framework is evaluated on degraded medical images using quantitative metrics, including Peak Signal-to-Noise Ratio (PSNR) and Structural Similarity Index Measure (SSIM), together with qualitative analysis of structural and detail preservation. Preliminary experimental results indicate that incorporating PDE-inspired guidance into a lightweight restoration architecture can improve image reconstruction quality while preserving the computational advantages of compact deep learning models. These findings highlight the potential of combining mathematical regularization with lightweight deep learning as an effective strategy for medical image restoration. The study also provides a basis for further investigation of robust restoration frameworks capable of handling more diverse and challenging degradation conditions.

Keywords: Medical image restoration, Image denoising, Lightweight deep learning, Partial differential equations, PDE-inspired regularization, Hybrid deep learning

Domain-Adversarial Deep Learning for Robust Glaucoma Detection Across Heterogeneous Fundus Image Datasets

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Abstract:

Automated glaucoma detection from fundus images has achieved promising performance with deep learning. However, the generalization of these models remains challenging because fundus datasets may exhibit substantial domain shifts caused by differences in acquisition devices, image characteristics, populations, and annotation protocols. This work proposes a domain-adversarial deep learning framework for robust glaucoma detection across heterogeneous fundus datasets. Three publicly available datasets, ORIGA, REFUGE, and G1020, are integrated within a unified learning framework using their ground-truth diagnostic labels. The proposed pipeline combines optic disc and optic cup segmentation and region-of-interest processing with deep feature extraction for binary glaucoma classification. To reduce dataset-specific bias, a Domain-Adversarial Neural Network (DANN) is incorporated through a Gradient Reversal Layer (GRL) and a domain classifier. The resulting adversarial training encourages the feature extractor to preserve glaucoma-discriminative information while learning representations that are less dependent on the dataset of origin. The study includes 2,470 labeled fundus images, comprising 1,926 non-glaucomatous and 544 glaucomatous cases. The proposed approach is evaluated against a conventional non-adversarial baseline using classification and domain-generalization criteria. The experiments are designed to investigate whether domain-adversarial representation learning can improve the robustness and cross-dataset generalization of automated glaucoma detection.

Keywords: Glaucoma detection, fundus images, deep learning, domain adaptation, domain-adversarial learning, DANN, medical image analysis

Cross-Lingual Disagreement for Training-Free Zero-Shot Image Recognition

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Abstract:

Multilingual vision-language models enable zero-shot image recognition using prompts written in different languages, yet equivalent prompts do not always produce consistent predictions. This work investigates cross-lingual prediction disagreement as a reliability signal for zero-shot image classification. We introduce XRoute, a training-free approach that measures the agreement, confidence, and prediction stability of language-specific outputs and uses these signals to select or combine the most reliable predictions for each image. Unlike conventional multilingual prompt ensembling, XRoute does not assume that all languages contribute equally and requires no fine-tuning, additional supervision, or model modification. Experiments across multiple image classification datasets and languages evaluate recognition performance and examine the relationship between cross-lingual disagreement and prediction errors. This analysis assesses whether language disagreement can serve as a simple and practical reliability signal for multilingual vision-language models at inference time.

Keywords: Zero-Shot Image Recognition, Vision-Language Models, Multilingual Vision-Language Models, Cross-Lingual Disagreement, Training-Free Inference, Uncertainty Estimation, Adaptive Language Routing, Image Classification

Sign Language Recognition and Translation using MediaPipe and LSTM-Based Deep Learning

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Abstract:

Sign language is a vital communication medium for the deaf community, yet the gap between signers and non-signers remains a significant challenge. This study presents a robust and lightweight Sign Language Recognition (SLR) system integrating MediaPipe with Convolutional Neural Networks (CNN) and Long Short-Term Memory (LSTM) networks. MediaPipe extracts precise landmarks from the hands, face, and body, which are processed by a CNN to capture spatial features and an LSTM to model temporal dependencies. The proposed approach effectively handles challenges such as hand occlusion, motion variation, and computational efficiency, enabling real-time performance. Experimental evaluation on the Moroccan Sign Language (MoSL) dataset demonstrates strong performance, achieving 98.2% validation accuracy, 97.8% F1-score, and 97.8% precision across 1,514 isolated gestures. These results highlight the system's robustness and potential for enhancing accessibility and communication between the hearing and non-hearing communities.

Keywords: Sign Language Recognition (SLR), Moroccan Sign Language (MoSL), MediaPipe, Convolutional Neural Networks (CNN), Long Short-Term Memory (LSTM)

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Multimodal Evolutionary Algorithms for Visual Recognition: From Passive Perception to Active Intelligence

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Abstract:

In this article, we present first the growth of visual recognition approaches toward agent-based hybrid systems, we review their foundations, inherent limitations, and key associated scientific challenges. Then, we adopt the analysis shortcomings and unresolved issues emerging during this transitional phase, while offering insights and perspectives to guide future research directions. Four implementation phases of current systems have built our observation; for each phase, the results obtained are presented together with the structural limitations encountered, classified according to scientific bottlenecks. The value of this article lies in its Synthesis section, which presents seven open scientific debates each articulating opposing positions, concrete proposals, and questions without an established answer on fundamental issues: the nature of "understanding" in visual AI, the resolution of the symbolic/sub-symbolic gap, the legitimacy of LLM based explainability, autonomy versus regulatory oversight, the measurement of agentic performance, systemic biases in biometric systems, and the epistemological implications of an autonomous visual AI.

Keywords: Visual recognition, BDI agents, Deep Learning, Decisional autonomy, Scientific debates

Dual-Backbone Hybrid Framework for Pneumonia Detection at the Edge

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Abstract:

Automated pneumonia detection from chest X-ray images can support computer-aided clinical screening, but deploying deep learning models on resource-constrained devices remains challenging because of their computational and memory requirements. This paper presents a lightweight hybrid framework that combines frozen EfficientNet-B0 and EfficientNet-B2 backbones with an RBF-kernel Support Vector Machine. The extracted representations are concatenated and refined using variance-based feature selection before classification. Experiments on the PneumoniaMNIST benchmark show that the best-performing configuration achieves 89.58% test accuracy, a pneumonia recall of 96%, and an F1-score of 0.88. The approach avoids end-to-end fine-tuning and reduces the training burden by decoupling feature extraction from classifier learning. Although the experiments were conducted on dual NVIDIA T4 GPUs, the modular design is intended to facilitate deployment on resource-constrained edge platforms. Direct measurements on physical edge hardware remain part of our future work. These results indicate that frozen CNN features combined with classical machine learning can provide a practical and competitive approach for lightweight medical image classification.

Keywords: Edge AI, Medical Image Classification, PneumoniaMNIST, Feature Fusion, Support Vector Machine, EfficientNet

Attention-Guided Frequency-Aware GAN for Blind Image Deblurring

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Abstract:

Blind image deblurring seeks to restore a sharp image from a blurry observation with no prior knowledge of the blur kernel, and remains one of the most ill-posed inverse problems in computer vision. GAN-based restorers such as DeblurGAN-v2 apply their convolutional filters uniformly across the image and supervise training only in the pixel and VGG-feature domains, although real motion blur is spatially non-uniform and acts as a low-pass filter that suppresses high spatial frequencies. This work investigates AF-DeblurGAN (Attention-guided Frequency-aware DeblurGAN), which integrates a CBAM channel-and-spatial attention module into every residual block of a four-level FPN generator and adds an explicit FFT loss to the hybrid objective, alongside MSE, VGG perceptual, SSIM and WGAN-GP terms ($\lambda_{MSE} = 10$, $\lambda_{perc} = 1$, $\lambda_{FFT} = 0.1$, $\lambda_{SSIM} = 0.5$, $\lambda_{adv} = 0.001$). Two configurations, differing in encoder depth (5.5M vs 12.9M parameters), training length (200 vs 300 epochs) and the presence of the SSIM term, are trained on GoPro Large (2,103 training pairs; 1,111 test pairs evaluated exhaustively at full 720×1280 resolution). The final configuration reaches PSNR = 29.82 dB, surpassing DeblurGAN-v2 (29.55 dB) by +0.27 dB with $4.6\times$ fewer parameters (12.9M vs 60M), and DeepDeblur (29.08 dB) with $23\times$ fewer. SSIM rises from 0.817 to 0.845 between the two configurations but stays below DeblurGAN-v2 (0.934), confirming the perception-distortion tradeoff. Since depth, training length and the SSIM loss vary jointly, a controlled ablation remains necessary to isolate each component.

Keywords: Blind deblurring, GAN, DeblurGAN-v2, CBAM attention, FFT loss, WGAN-GP, PSNR, SSIM, GoPro

Change Detection Using Temporal Satellite Image Analysis with ViT

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Abstract:

Automatic change detection (CD) in high resolution remote sensing imagery plays a crucial role in modern urban planning, environmental monitoring, and disaster management. And despite the significant progress made by deep learning in this field, traditional convolutional neural networks (CNNs) is still suffer from structural limitations, due to their limited receptive fields. Which often fail to capture long range global context, resulting in fragmented edge detection and high false alarm rates. To address these limitations, this paper introduces CrossViT-CD, wish used an innovative tools, attention driven, two branch Vision transformer framework specifically optimized for detecting changes in bitemporal remote sensing data. By exploiting self-attention mechanisms and cross scale token interaction mechanisms. So the suggested architecture effectively models comprehensive spatio temporal dependencies and aligns features across different time stamps. Then leading to the successful suppression of irrelevant environmental noise such as seasonal variations and lighting changes. The framework was rigorously evaluated on the extremely competitive benchmark LEVIR-CD dataset over 200 epochs. The experimental results show that CrossViT-CD performs strongly. With an overall accuracy (OA) of 98.50%, an intersection over union (IoU) of 75.49% and an F1 score of 85.95%. These results represent a significant improvement over classical fully convolutional twin networks, such as a 2.55% gain in F1-Score compared to the standard baseline fully convolutional early fusion (FC-EF). Also the results show that mutual attention mechanisms are a highly effective model for feature integration, offering a robust and extensible foundation for next generation automated earth observation data processing.

Keywords: Change Detection, Remote Sensing, Vision Transformers, Cross-Attention, LEVIR-CD

Enhancing Mammography Classification with Multimodal Information Fusion

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Abstract:

Mammography classification remains challenging because benign and malignant findings can share subtle visual patterns. In clinical practice, radiologists also rely on metadata and radiological reports in addition to images. This study therefore investigates multimodal fusion for three-class mammography classification using the DMID dataset. A multimodal framework was developed by combining mammographic images, acquisition metadata, and radiological reports. Original and cleaned reports were evaluated to assess the impact of diagnosis-revealing information, while intermediate and late fusion strategies were compared under the same experimental conditions. The results showed that intermediate fusion provided the best overall classification performance and predictive reliability. Cleaning the reports reduced performance, although the remaining descriptive information continued to provide useful complementary information. Overall, the findings highlight the value of multimodal fusion and the importance of carefully controlling diagnosis-revealing information in radiological reports.

Keywords: mammography classification, multimodal fusion, radiological reports, intermediate fusion, late fusion, DMID dataset

A Novel Data-Driven Architecture for Enhanced Diagnostic Accuracy in Benchmark Medical Image Datasets

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Abstract:

Medical image analysis plays a pivotal role in modern computer-aided diagnosis, enabling early and accurate detection of pathologies. However, traditional machine learning models often face significant challenges when dealing with high-dimensional data, noise, and complex spatial patterns inherent in medical imaging. This paper proposes a novel data-driven architecture designed to optimize feature representation and enhance diagnostic accuracy across standard benchmark medical image datasets. The proposed framework integrates advanced deep learning feature extraction techniques with optimized classification strategies to minimize computational overhead while maximizing generalization performance. Extensive experimental evaluations conducted on benchmark datasets demonstrate that the proposed architecture achieves superior performance in terms of classification accuracy, sensitivity, and specificity compared to conventional baseline models. These findings highlight the robustness and reliability of the proposed approach, offering a powerful computational tool for clinical decision support systems. Future work will focus on extending this framework to multi-modal medical data and exploring its integration into real-time diagnostic workflows.

Keywords: Data-Driven Architecture, Medical Image Analysis, Deep Learning, Diagnostic Accuracy, Classification, Benchmark Datasets

An Optimized Transfer Learning Framework with PSO-Tuned KNN for Early-Stage Parkinson's Disease Detection

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Abstract:

Automated early-stage detection of Parkinson's disease (PD) remains one of the most challenging tasks in medical diagnosis, as patients in the early stages often exhibit symptoms very similar to those of healthy individuals. Although numerous approaches have been proposed to improve diagnostic accuracy, reliable and non-invasive early detection of PD remains difficult. In this work, we propose an optimized hybrid framework combining multi-model transfer learning and PSO-based classifier optimization for early PD detection from handwriting images. Specifically, three ImageNet-pretrained CNN architectures: ResNet-50, VGG-19, and Inception v3 are used as frozen feature extractors, and their deep feature maps are concatenated to form a rich and diverse feature representation. Particle Swarm Optimization (PSO) is subsequently used to tune the hyperparameters of the k-Nearest Neighbors (KNN) algorithm. The framework is evaluated on the publicly available NewHandPD dataset, which comprises handwriting samples from both healthy individuals and PD patients across three drawing tasks: spiral, meander, and circle. Experimental results demonstrate that the proposed framework achieves classification accuracies of 93.33% on spiral drawings, 96.43% on circle drawings, and 96.43% on meander patterns, with an overall accuracy of 96.92% and an AUC of 0.96, outperforming several existing state-of-the-art approaches for early PD diagnosis. Statistical significance tests further confirm the robustness of the proposed method, with highly significant improvements ($p < 0.01$) over the majority of competing methods.

Keywords: Parkinson's Disease Detection, Transfer Learning, Handwriting Analysis, Particle Swarm Optimization, k-Nearest Neighbors, Feature Fusion

Deep Learning for Retinal Vessel Segmentation: A Systematic Search and Critical Review of Datasets, Evaluation Protocols, and Reproducibility Challenges

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Abstract:

Accurate segmentation of retinal blood vessels in color fundus images supports quantitative vascular analysis and computer-aided assessment of ocular and systemic disease. Although deep learning has substantially advanced this task, published performance figures remain difficult to interpret because they are influenced not only by network design but also by data partitioning, image resolution, preprocessing, field-of-view masking, threshold calibration, and post-processing. To examine these sources of variation, we conducted a systematic search of research published between January 2015 and September 2026 across Scopus, IEEE Xplore, PubMed, SpringerLink, and ScienceDirect, following PRISMA 2020 terminology for the identification and deduplication stages. The search yielded 3,260 records; removal of 1,088 duplicates left 2,172 unique records. A conservative title-and-metadata audit excluded 493 clearly ineligible records and retained 1,679 for further relevance assessment. Rather than treating reported benchmark scores as directly comparable, this review critically considers the interplay between model architecture, dataset characteristics, and evaluation practices. It examines U-Net variants, attention- and Transformer-based networks, topology-aware learning, lightweight and state-space architectures, domain generalization, semi-supervised methods, and uncertainty-aware approaches. The review highlights the need for consistent experimental protocols, transparent reporting of methodological choices, and external validation to establish whether apparent performance gains translate into robust, reproducible retinal vessel segmentation

Oral Session 3. Partial Differential Equations and Scientific Computing

MATHEMATICAL ANALYSIS OF SOME THREE-DIMENSIONAL MODELS IN FLUID MECHANICS

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Abstract:

We study a family of dissipative active scalar equations with velocity fields coupled through higher-order multiplier operators. Focusing on sub-critical values for fractional diffusion, we establish global well-posedness for solutions with small initial data in a framework based on Fourier transforms, specifically Fourier-Besov-Morrey spaces. The smallness condition is tied to the weak norm of this space, allowing for initial data with large L^2 -norms to be considered. We derive self-similar solutions depending on the homogeneity of the initial data and couplings and demonstrate that the solutions become asymptotically self-similar at large times. Our unified approach applies to various active scalar PDEs, including 1D models of dislocation dynamics in crystals, Burgers' equation, the 2D vorticity equation, 2D generalized SQG, and 3D magneto-geostrophic equations.

Keywords: Active scalar equations, global well-posedness, self-similar solutions

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Nontrivial Weak Solutions for Steklov Problems with $(p(x), q(x))$ -Laplacian

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Abstract:

In this study, we demonstrate that the following Steklov boundary value problem involving $(p(x), q(x))$ -Laplacian has nontrivial weak solutions in diverse situations.

$$\begin{cases} \Delta_{p(x)}u + \Delta_{q(x)}u = 0 & \text{in } \Omega, \\ (|\nabla u|^{p(x)-2} + |\nabla u|^{q(x)-2}) \frac{\partial u}{\partial \nu} = g(x, u) - |u|^{p(x)-2}u - |u|^{q(x)-2}u & \text{on } \partial\Omega. \end{cases}$$

The mountain pass theorem and a conventional Weierstrass type theorem are the basis for the existence results. We used the symmetric mountain pass theorem to derive our multiplicity results.

Keywords: Mountain Pass Theorem, Steklov eigenvalue problem, $(p(x), q(x))$ -Laplacian operator, Weierstrass type theorem

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Existence of Solutions for a Class of Schrödinger Systems in Orlicz-Sobolev Spaces

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Abstract:

In this work, we established a Schrödinger system involving $\omega(\cdot)$ -Laplacian operator. Our results are extensions and generalizations of some existing results in the literature.

Keywords: Quasi-linear Schrödinger system, Orlicz-Sobolev spaces, $\omega(\cdot)$ -Laplacian operator, Mountain pass theorem, Weak solution

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Existence of Capacity Solutions for a Nonlinear Coupled Thermoelastic Semiconductor System in Orlicz-Sobolev Spaces

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Abstract:

We investigate a nonlinear coupled thermoelastic semiconductor system within the framework of Orlicz-Sobolev spaces. The model consists of an elliptic equation for the electrostatic potential, drift-diffusion equations for electron and hole densities, a parabolic heat equation, and a hyperbolic elasticity system describing mechanical displacement. The nonlinear diffusion is governed by a Young function Φ satisfying the Δ_2 -condition, allowing for nonstandard growth behaviors, including exponential and logarithmic types, beyond the scope of the classical L^p setting. By combining regularization techniques, uniform modular a priori estimates, a generalized Aubin-Lions compactness lemma in Orlicz spaces, and a monotonicity argument based on Minty's method, we establish the existence of at least one capacity solution. Under additional monotonicity and Lipschitz assumptions on the nonlinear terms, conditional uniqueness is also addressed. Numerical simulations together with two-dimensional and three-dimensional visualizations are provided to illustrate the coupled electrostatic, thermal, and mechanical dynamics predicted by the theoretical analysis. Finally, possible extensions to fractional Orlicz-Sobolev spaces are briefly discussed.

Pseudomonotone Operator Methods for Nonmonotone Discrete Robin Problems

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Abstract:

We investigate a second-order discrete Robin problem involving the discrete p -Laplacian. The main difficulty arises from the possible nonmonotonicity of both the interior reaction term and the nonlinear Robin boundary perturbation. In contrast to variational approaches based on critical point geometry or Ricceri–Bonanno type results, we adopt a pseudomonotone operator framework. The coercivity of the principal part is combined with the finite-dimensional compactness of the nonlinear interior and boundary terms, allowing us to establish the existence of a weak solution under weak one-sided coercivity assumptions. Moreover, uniqueness is obtained under additional one-sided monotonicity conditions.

Keywords: discrete p -Laplacian, discrete Robin problem, nonmonotone problems, pseudomonotone operators, weak solutions, one-sided monotonicity

WHC-PINN for Solving Compressible Euler Equations with Shock Waves

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Abstract:

In this work, we present the Weighted-loss and Hard-Constraint Physics-Informed Neural Network (WHC-PINN) framework for solving the quasi-one-dimensional compressible Euler equations involving shock waves and contact discontinuities. The implementation combines a gradient-based weighting strategy for the PDE residual with specific physical constraints to improve the treatment of discontinuities. Two fundamental problems are considered: the Sod Riemann problem and the converging-diverging nozzle flow, which represent initial-value and boundary-value problems, respectively. For the Riemann problem, a conservation-aware loss is incorporated to enforce the integral conservation of mass, momentum, and energy, while a hard pressure constraint is applied to the nozzle problem. The neural network is trained using a two-stage optimization procedure based on Adam and L-BFGS. The numerical results are compared with analytical reference solutions in terms of density, velocity, temperature, pressure, and Mach number. The obtained results demonstrate the practical behavior of the WHC-PINN framework in capturing shock waves and resolving compressible flow features, providing insight into its implementation for hyperbolic problems.

Keywords: Partial differential equations, Numerical analysis, Physics-informed neural networks, Compressible flow, Euler equations, Shock capturing, Riemann problem, Converging-diverging nozzle, Scientific computing

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L'effet d'ordre inférieur pour les équations elliptiques quasi-linéaires anisotropes avec non-linéarité singulière

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Abstract:

Cet article s'intéresse à l'étude de l'existence ainsi que de l'unicité des solutions non négatives d'une famille d'équations elliptiques quasi-linéaires anisotropes comportant une non-linéarité singulière. Plus précisément, les auteurs considèrent le problème suivant :

$$-\Delta_p^H u + |u|^{r-1}u = \frac{f(x)}{u^\theta} \quad \text{dans } \Omega, \quad (3.1)$$

où $\Omega \subset \mathbb{R}^N$ est un domaine borné régulier, avec $N > p > 1$, $r > 1$, $0 < \theta < 1$, $N \geq 2$, et une fonction $f \geq 0$ non identiquement nulle dans Ω , $f \in L^1(\Omega)$.

L'opérateur Δ_p^H désigne le p -Laplacien anisotrope défini par

$$\Delta_p^H u = \operatorname{div} (H^{p-1}(\nabla u) \nabla H(\nabla u)).$$

Keywords: elliptic equations, quasi-linear anisotropic, singular nonlinearity, lower order term, p -Laplacian anisotropic

On L^p -Regularity for an Elliptic Equation in an Unbounded Domain

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Abstract:

We investigate the L^p -regularity of an elliptic boundary-value problem posed on the unbounded strip $(0, +\infty) \times (0, 1)$. The problem is reformulated in an abstract operator framework on the Bochner space $L^p(0, +\infty; L^q(0, 1))$, with $p, q \in (1, \infty)$, by decomposing the differential operator into two commuting closed operators corresponding, respectively, to the unbounded and bounded spatial directions. Under a suitable resolvent estimate for the realization of the one-dimensional elliptic operator, the Dore–Venni theorem is employed to establish the closedness and maximal regularity of their sum. As a consequence, for every source term $f \in L^p(0, +\infty; L^q(0, 1))$, the boundary-value problem under consideration admits a unique strict solution. Moreover, the solution satisfies maximal mixed Sobolev regularity; more precisely, it has second-order Sobolev regularity with respect to both variables and satisfies the prescribed homogeneous boundary conditions. These results establish a functional-analytic characterization of existence, uniqueness, and L^p -maximal regularity for elliptic equations in unbounded domains.

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On the Viscosity Selection Properties of Neural Solvers for Hamilton-Jacobi-Bellman Equations in Finance

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Abstract:

We study the numerical approximation of value functions of stochastic control and optimal stopping problems arising in finance, governed by Hamilton-Jacobi-Bellman equations of obstacle type $\min(-\partial_t u - Lu, u - \psi) = 0$, where L is the generator of the underlying diffusion and ψ an obstacle encoding the control. Such solutions lose C^2 regularity across the free boundary and must be understood in the viscosity sense of Crandall and Lions, the unique notion among infinitely many almost-everywhere solutions that characterises the value function. Monotone schemes converge to it within the Barles-Souganidis framework, which provides a certified reference. Neural approximations enjoy no comparable guarantee, and the reasons differ across families: residual-based methods such as physics-informed neural networks minimise a symmetric loss that does not encode the order structure underlying viscosity selection, whereas probabilistic methods such as deep BSDE schemes target an object identified with the viscosity solution through the nonlinear Feynman-Kac representation, leaving only the optimisation step unguaranteed. Existing approximation results establish the expressive power of deep networks for parabolic HJB equations, but under non-degeneracy assumptions where the value function is classical, and say nothing about which solution a trained network reaches. We therefore benchmark several neural approximations against a convergent scheme on a low-dimensional financial model admitting a reliable reference. Beyond the global L^∞ error, we quantify free-boundary localisation and realised policy performance, and we let the diffusion degenerate towards the first-order regime, where selection is most at stake.

Keywords: Hamilton-Jacobi-Bellman equations, viscosity solutions, neural solvers, optimal stopping, stochastic control, physics-informed neural networks, deep BSDE schemes, free boundary

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Very Weak Solutions for Schrödinger Equations with Singular Potentials in Jacobi–Dunkl Analysis

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Abstract:

We study a Jacobi–Dunkl Schrödinger equation with a compactly supported singular potential and distributional initial data. The main difficulty is that the product between a singular potential and a distributional solution is generally not defined in the classical theory of distributions. This motivates the development of a spectral very weak solution framework capable of treating singular coefficients while preserving the underlying Jacobi–Dunkl structure. Our approach combines mollification regularization with Jacobi–Dunkl spectral analysis. For suitable regularizations of the potential and the initial datum, we consider the associated regularized Cauchy problems in the weighted space $L^2_{\alpha,\beta}(\mathbb{R})$. The spectral framework is based on the harmonic analysis of Jacobi–Dunkl operators, while the very weak formulation follows the regularization philosophy developed for equations with singular coefficients and, more recently, for Schrödinger-type equations with singular coefficients. Under suitable assumptions, the regularized Hamiltonians are self-adjoint, yielding unitary Schrödinger propagators and uniform-in-time $L^2_{\alpha,\beta}$ -estimates. These estimates imply the moderateness of the corresponding solution nets and provide the existence of $L^2_{\alpha,\beta}$ -valued very weak solutions. We further establish uniqueness modulo negligible nets with respect to equivalent admissible regularizations and prove consistency with the classical $L^2_{\alpha,\beta}$ -solution whenever the potential and initial datum possess sufficient regularity. The framework provides a unified spectral treatment of singular Schrödinger equations in Jacobi–Dunkl harmonic analysis, with potential applications to Dirac-type potentials, point interactions, and distributional initial data.

Keywords: Jacobi–Dunkl analysis, Schrödinger equations, singular potentials, very weak solutions, distributional initial data, spectral theory, Colombeau-type regularization

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Hybrid Physics Guided Machine Learning-Continuous Data Assimilation for Dissipative Partial Differential Equations: General Theory and Applications to the Two-Dimensional Navier–Stokes and Bénard Systems

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Abstract:

We develop a physics-guided framework for continuous data assimilation (CDA) in dissipative partial differential equations. The framework combines partial-in-space observations with a learned representation of the unresolved, incomplete, or computationally expensive forecast dynamics. The continuous feedback mechanism is of Azouani–Olson–Titi type and remains the central synchronization device: it continuously corrects the forecast by comparing observed coarse scales of the assimilated state with those of the reference state. We prove an abstract tracking estimate showing exponential convergence to a neighborhood of the true solution, with an explicit error level determined by the accuracy and stability of the learned dynamics and by observational noise. We then formulate the theory for the two-dimensional incompressible Navier–Stokes equations and for the two-dimensional Bénard convection system. The results identify conditions on the observation resolution, nudging gain, and learned-dynamics residual that preserve long-time tracking. This provides a rigorous mathematical basis for combining CDA with physics-guided machine learning in dissipative fluid systems.

Keywords: Continuous data assimilation, Azouani–Olson–Titi feedback, physics-guided machine learning, dissipative PDEs, Navier–Stokes equations, Bénard convection, synchronization

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Analysis and Numerical Approximation of the First Eigenvalue for Variable-Exponent Double-Phase Problems with Robin Boundary Conditions

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Abstract:

In this work, we investigate Robin eigenvalue problems governed by variable-exponent double-phase operators and explore their numerical approximation using Physics-Informed Neural Networks (PINNs). We first establish the existence and uniqueness of solutions within a suitable variational framework. We then develop a PINN-based computational method for approximating the corresponding eigenfunctions and first eigenvalue. The performance of the proposed approach is finally illustrated through several numerical experiments conducted under various configurations.

More precisely, we study the following problem:

$$\begin{cases} -\operatorname{div} \left(|\nabla u|^{p(x)-2} \nabla u + \mu(x) |\nabla u|^{q(x)-2} \nabla u \right) = \lambda f(x, u), & \text{in } \Omega, \\ \left(|\nabla u|^{p(x)-2} \nabla u + \mu(x) |\nabla u|^{q(x)-2} \nabla u \right) \cdot \frac{\partial u}{\partial \vec{n}} + \beta(x) |u|^{r(x)-2} u = g, & \text{on } \partial\Omega, \end{cases} \quad (P_\lambda)$$

under suitable assumptions on the functions and parameters involved.

Keywords: Double Phase Problems, Variable Exponents, Robin Boundary Conditions, Physics-Informed Neural Networks

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From Conservation Laws to Balance Laws: Well-Balanced WE-PINNs for Shallow Water Flows over Complex Bathymetries

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Abstract:

Physics-Informed Neural Networks (PINNs) face fundamental difficulties when applied to nonlinear hyperbolic problems involving shocks, source terms, and non-trivial steady states. In previous work, we introduced Weak and Entropy Physics-Informed Neural Networks (WE-PINNs), a mesh-free framework for hyperbolic conservation laws in which conservation is enforced through weak space-time control-volume identities, avoiding differentiation of the neural approximation across discontinuities. In this work, we extend the WE-PINN framework from homogeneous conservation laws to hyperbolic balance laws, focusing on the shallow water equations over variable bathymetry. The main challenge is to preserve exactly the delicate equilibrium between flux gradients and geometric source terms, while also allowing stationary discontinuities generated by discontinuous bottom topographies. Using the free-surface elevation as a primary variable, we construct a weak formulation in which the lake-at-rest equilibrium is preserved exactly. The source contribution is evaluated through an integration-by-parts formulation that does not differentiate the neural network, together with a local correction of the bed slope ensuring exact discrete cancellation independently of the control-volume sampling and quadrature order. For discontinuous bathymetries, the computational domain is automatically decomposed at detected bed steps, and separate neural representations are coupled through appropriate stationary transition conditions. We show that the resulting discrete residual vanishes identically at the lake-at-rest equilibrium, making this steady state an exact global minimizer of the training objective. Numerical experiments involving smooth, irregular, and discontinuous bathymetries, including Riemann problems over bottom steps, demonstrate the ability of the proposed approach to preserve steady states while accurately capturing transient shallow water dynamics. These results extend the WE-PINN methodology toward structure-preserving neural solvers for nonlinear hyperbolic balance laws with complex geometries and discontinuous data.

Keywords: Physics-Informed Neural Networks, WE-PINNs, shallow water equations, hyperbolic balance laws, well-balanced methods, weak formulation, discontinuous bathymetry

Normalized solutions for a capillary-type equation with variable exponent $\varrho(\cdot)$ and critical Hardy potential

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Abstract:

In this paper, we study the existence and concentration of normalized solutions for a capillary-type elliptic equation with variable exponent $\varrho(\cdot)$, a bounded potential U , a critical Hardy potential and a logarithmic nonlinearity, under a modular mass constraint. Since the exponent varies, neither the amplitude scaling nor the Gagliardo–Nirenberg inequality is available; we replace the latter by an interpolation inequality in variable exponent Lebesgue spaces, and we establish a modular Hardy inequality adapted to the variable exponent setting, together with a Sobolev embedding whose constants are uniform under dilation of the exponent. We prove that, for α large, γ small and ε small, the problem admits at least κ distinct solutions with negative Lagrange multipliers, positive away from the pole, concentrating at the κ minima of a level function which combines the potential and the exponent and reduces to U in the constant-exponent case. The concentration occurs at two different scales: away from the pole the solutions decay exponentially, whereas at the pole the Hardy singularity pins the profile at the scale ε and the corresponding limiting level lies strictly below that of the other concentration points. We also show that the capillary structure degenerates in the semiclassical limit along concentrating families, but not uniformly, which is what governs the level seen by escaping sequences.

Keywords: Normalized solutions, Variable exponent, Capillary operator, Hardy potential, Logarithmic nonlinearity, Concentration phenomena, Musielak–Orlicz–Sobolev spaces, Semiclassical limit, Modular constraint

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Accounting for skin effect in the Helmholtz problem

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Abstract:

In this talk, we account for the skin effect in the Helmholtz transmission problem.

$$\begin{cases} \Delta u_j + k_j^2 u_j = 0 & \text{in } O_j, \\ \partial_n u_3 - ik_3 u_3 = h & \text{on } \Gamma_3, \\ [u] = [\alpha \partial_n u] = 0 & \text{on } \Gamma_1 \cup \Gamma_2. \end{cases}$$

We derive approximate boundary transmission conditions via an asymptotic expansion in the characteristic size of the thin-layer O_2 . We show the well-posedness of the achieved equation thanks to the study of a saddle-point problem [1]. The theoretical results are illustrated through numerical simulations with finite elements in FreeFEM++ [2]. Finally, the model is applied to address practical challenges in medical imaging and non-destructive testing for aeronautics.

Keywords: Partial differential equations, Numerical analysis, Finite elements, Inverse problems, Asymptotic development

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Existence and stability of numerical radial nonnegative solutions for a class of semipositone elliptic problems in a ball using Shooting method

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Abstract:

In this paper, we investigate the existence of numerical nonnegative radial solutions for a semipositone semilinear elliptic boundary value problem of the form $-\Delta u = \lambda f(u)$ in Ω , $u = 0$ on $\partial\Omega$, where Ω is the unit ball in $\mathbb{R}^n$, $n \geq 2$, and $\lambda > 0$ is a parameter. The nonlinearity satisfies the semipositone condition $f(0) < 0$, $f \in C(\mathbb{R}^+, \mathbb{R})$, has exactly one positive zero and concave, we analyze numerically the structure of nonnegative radial solutions. Using radial symmetry, the problem is reduced to an ordinary differential equation. We apply the shooting method and study the qualitative behavior of the associated initial value problem.

Keywords: Partial differential equations, Numerical analysis, Semipositone problem, Radial solutions, Elliptic equation, Shooting method

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A Nonlinear Algorithm Based on a Variable-Exponent System for Multiplicative Gamma Noise Removal

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Abstract:

In this paper, we introduce a novel nonlinear reaction–diffusion system for the restoration of images corrupted by multiplicative Gamma noise. The proposed framework combines an H^{-1} -based fidelity term with a variable γ -Laplacian operator and incorporates a regularized Perona–Malik (PM) equation to provide an effective preliminary smoothing of noisy images. For the numerical implementation, an explicit finite difference scheme is developed to approximate the proposed system. The effectiveness of the approach is assessed through numerical experiments on both textured and non-textured images. Experimental results highlight the ability of the proposed model to efficiently suppress multiplicative noise while preserving significant image structures and fine textural details, demonstrating its competitive performance compared with existing image restoration methods.

Nonnegative periodic weak solutions in variable-exponent parabolic equations.

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Abstract:

This work investigates a parabolic equation governed by a degenerate $p(x)$ -Laplacian, with strongly nonlinear source term and Dirichlet boundary conditions. The variable exponent in the differential operator gives the equation a setting in which its behavior may vary across the domain. We establish the existence of a nonnegative periodic weak solution to this problem. The argument is based on the Leray-Schauder topological degree.

Mean Curvature Equations: Existence, Regularity, and Analytical Techniques

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Abstract:

Mean curvature equations are important nonlinear elliptic problems arising in differential geometry and geometric analysis. In this presentation, we introduce the mean curvature operator and discuss the existence and regularity of solutions. We also present some basic analytical techniques used to study these equations and their applications.

Keywords: Mean curvature, Nonlinear elliptic equations, Existence of solutions, Regularity, Partial differential equations

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Oral Session 4. Optimization,
Numerical Algorithms and Fuzzy
Decision Models

Un algorithme Dragonfly hybride basé sur une recherche locale à voisinages multiples pour le Job Shop Scheduling

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Abstract:

The Job Shop Scheduling Problem (JSSP) remains one of the most challenging combinatorial optimization problems in production scheduling due to its NP-hard nature and the rapid growth of the search space as problem size increases. Despite extensive research on meta-heuristic approaches, achieving an effective balance between solution quality, convergence speed, and computational efficiency remains a major challenge, particularly in manufacturing environments where scheduling decisions may need to be produced within limited computational time. To address this issue, this study proposes DA+MNLS, a hybrid optimization approach that integrates the population-based exploration capabilities of the Dragonfly Algorithm (DA) with an intensive Multi-Neighborhood Local Search (MNLS) mechanism. The proposed MNLS incorporates five complementary neighborhood operators: adjacent swap, critical-path-targeted swap, insertion, subsequence reversal, and block swap. These operators are designed to explore different structures of the scheduling solution space and strengthen the exploitation capability of the algorithm. An adaptive depth-control mechanism dynamically adjusts the search effort according to the observed effectiveness of the neighborhood operators, while a strategic restart mechanism is incorporated to mitigate premature convergence and population stagnation. The proposed approach is evaluated on standard Fisher-Thompson and Lawrence JSSP benchmark instances and compared with representative scheduling approaches. The experimental evaluation considers solution quality, computational time, convergence behavior, and robustness across benchmark instances of different sizes. This framework aims to provide an effective balance between global diversification and problem-specific local intensification for solving challenging JSSP instances.

Keywords: Job Shop Scheduling, Dragonfly Algorithm, Multi-Neighborhood Local Search, Hybrid Optimization, Combinatorial Optimization

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Crisp-Fuzzy decomposition method for solving fuzzy differential and fuzzy integro-differential equations

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Abstract:

In this paper, we introduce the Crisp-Fuzzy decomposition of a fuzzy number and fuzzy valued functions, we exhibit its applications for fuzzy differentiability and fuzzy integrability. Then, we present a new method, called Crisp-Fuzzy decomposition method, to solve fuzzy differential equations, fuzzy differential systems and fuzzy integro-differential equations. Therefore, we prove the efficiency of this method by giving some numerical examples.

Keywords: Crisp-Fuzzy decomposition, fuzzy differential equations, fuzzy integro-differential equations, fuzzy differentiability, fuzzy integrability

Dynamic Momentum Optimization: Lyapunov Stability and Implicit Regularization

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Abstract:

Momentum is a core component of gradient-based optimization. However, traditional methods rely on a fixed momentum parameter, which can lead to instability, oscillatory dynamics, and poor generalization in complex loss landscapes. In this work, we propose an optimizer that dynamically adjusts its momentum coefficient based on the local geometry of the objective function. To support this approach, we provide a rigorous Lyapunov-based energy analysis, establishing strict energy dissipation and convergence guarantees for the dynamic-coefficient setting, extending existing mathematical frameworks for momentum-based methods. We evaluate the proposed method across two different scales. On a deterministic, ill-conditioned benchmark, the adaptive scheme successfully mitigates the severe oscillations typically caused by high static momentum. Furthermore, in a stochastic, large-scale deep learning setting, our method demonstrates more stable training dynamics and improves test accuracy compared to static baselines. These empirical results suggest that dynamic momentum acts as an implicit regularizer. By navigating towards wider minima, the optimizer trades some training precision for significantly better generalization at test time.

Keywords: Dynamic Momentum, Convex and Non-Convex Optimization, Lyapunov Stability, Implicit Regularization, Deep Learning

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Markov Decision Processes with discounted triangular intuitionistic fuzzy costs

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Abstract:

This paper develops a Markov Decision Process (MDPs) framework for sequential decision-making under uncertain and imprecise costs represented by triangular intuitionistic fuzzy numbers (TIFNs). Unlike ordinary MDPs, which assume precise costs, the proposed model incorporates membership, non-membership, and hesitation information into the cost structure. A discounted cumulative intuitionistic fuzzy cost criterion is formulated, and an extended Bellman optimality equation is established for infinite-horizon decision problems. To support optimization, appropriate arithmetic, ranking, and comparison operations for TIFNs are defined and integrated into the value-iteration procedure. Under suitable assumptions on the discount factor and operators, convergence and uniqueness of the optimal value function are investigated. A numerical experiment illustrates the proposed methodology, identifies the optimal policy, and examines the effects of discounting and uncertainty. Sensitivity analysis further demonstrates the models robustness and potential for practical decision-making under incomplete information in complex environments.

Keywords: Markov Decision Processes, Triangular Intuitionistic Fuzzy Numbers, Discounted Costs, Bellman Equation, Uncertainty in Decision-Making, Optimal Policy

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Inertial Algorithms for Constrained Convex Optimization with Applications to Olive Leaf Disease Classification

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Abstract:

We consider the problem of minimizing a smooth convex objective function over the set of minimizers of a differentiable convex function. To solve this hierarchical optimization problem, we propose an inertial gradient-based algorithm incorporating a penalization term. The inertial component exploits the history of the iterates and is designed to improve the convergence behavior of the method. Under a suitable condition involving the Fenchel conjugate of the constraint function, we first establish the weak convergence of the generated sequence to a solution of the considered optimization problem. We then prove the strong convergence of the objective function values toward the optimal value under a strong convexity assumption. The strong convergence analysis relies on a new Lyapunov approach, which provides the main tool for establishing the asymptotic properties of the proposed algorithm. Finally, the effectiveness of the proposed method is illustrated through numerical experiments on an image classification problem, with particular emphasis on olive leaf disease classification.

Keywords: Inertial forward-backward, Strong and weak convergence, Inverse problems, Classification, Gradient descent

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Limiting Behavior of Solutions of Optimal Control Problems Governed by Nonlinear Elliptic Boundary Value Problems in an Oscillating Thin Layer

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Abstract:

The aim of this paper is to study the limiting behavior of solutions of a class of optimal control problems governed by a nonlinear elliptic boundary value problems in a structure composed on an oscillating thin layer of thickness depending of the positive parameter ε which is intended to tend toward 0. With a cost functional type that depends on ε , we use the epiconvergence method to prove the optimality result which characterizes the limit optimal control problem.

Keywords: optimal control, nonlinear elliptic boundary value problems, oscillating thin layer, epiconvergence, limiting behavior

Minimizing DC Fractional Programs subject to DC Block-Separable Constraints

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Abstract:

We are interested in solving fractional problems whose objective function is defined as the ratio of the difference of convex (DC) functions subject to block-separable constraints that are also DC functions. In addition to its non-convexity, this type of problem is difficult to solve in a high-dimensional setting. To overcome these difficulties, we transform the fractional problem into a sequence of simple and convex subproblems by combining Dinkelbach's method, block coordinate descent methods, the proximal point method and DC programming techniques. The convergence of the proposed methods is established, and numerical experiments are provided to illustrate their performance.

Keywords: Fractional programming, DC functions, Dinkelbach's method, block coordinate descent method, proximal point method

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A study of (α, β) -intuitionistic fuzzy Orders

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Abstract:

In this work, we establish some properties of (α, β) -intuitionistic fuzzy order relations. Firstly, we prove that if $(\alpha, \beta) \in]0, 1[\times]0, 1[$ with $\alpha + \beta \leq 1$ and $(R_i)_{i \in I}$ is a family of (α, β) -intuitionistic fuzzy order relations on X , then the intuitionistic fuzzy relation $R = (\mu_R, \nu_R)$ defined by: $\mu_R(a, b) = \inf_{i \in I} \{\mu_{R_i}(a, b)\}$ and $\nu_R(a, b) = \sup_{i \in I} \{\nu_{R_i}(a, b)\}$ for every $(a, b) \in X \times X$ is an (α, β) -intuitionistic fuzzy order relation on X [7, Theorem 3.1]. Secondly, we show that if for every $i \in \{1, \dots, n\}$, $R_i = (\mu_{R_i}, \nu_{R_i})$ is compatible with the vector algebraic structures on X_i , so $V = (\mu_V, \nu_V)$ defined on the real vector space $X = \prod_{i=1}^n X_i$ by: $\mu_V((a_1, \dots, a_n), (b_1, \dots, b_n)) = \min_{i \in \{1, \dots, n\}} \{\mu_{R_i}(a_i, b_i)\}$ and $\nu_V((a_1, \dots, a_n), (b_1, \dots, b_n)) = \max_{i \in \{1, \dots, n\}} \{\nu_{R_i}(a_i, b_i)\}$ is an (α, β) -intuitionistic fuzzy order relation compatible with the vector algebraic structures on X [7, Theorem 4.2]. Finally, we present a characterization of a class of total (α, β) -intuitionistic fuzzy order relations compatibles with the usual algebraic structures on the real line $\mathbb{R}$ [7, Theorem 5.8].

Keywords: intuitionistic fuzzy set, (α, β) -intuitionistic fuzzy order relations

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Structured Hessian Initialization in L-BFGS for Regularized Inverse Diffusion Problems: Benefits and Limitations

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Abstract:

We study the role of the initial Hessian approximation in limited-memory BFGS for a regularized inverse diffusion problem, in which a spatially varying diffusivity is recovered from observations of the state. Within a single objective and a strictly like-for-like protocol—same data, initial guess, line search, memory size, stopping parameters, and iteration budget—we compare a scalar initialization with the structured, regularization-aware seed $\tau_k I + \lambda L^\top L$ used in structured L-BFGS methods. Three findings emerge. First, the structured seed is essentially neutral when the regularization weight is small and becomes clearly advantageous as it grows, substantially reducing the best reconstruction error. Second, this gain is realized at an intermediate iterate: the structured runs reach a more accurate reconstruction and then deteriorate by progressively over-smoothing a genuine narrow feature of the coefficient. This semi-convergence persists in the noise-free case, showing that it is driven by the regularization rather than by measurement noise. Third, two natural metric-based correctors, one residual-aware and one roughness-aware, fail to repair this behavior. The reason is structural: changing the initial Hessian seed modifies the optimization trajectory, but neither the regularized objective nor its stationary equations. Structured initialization is therefore useful, but it does not remove the need for a reliable stopping rule and a sound choice of the regularization parameter.

Keywords: L-BFGS, structured Hessian initialization, inverse diffusion problem, regularization, semi-convergence, PDE-constrained optimization

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Some properties of the Recursive Hermite-Birkhoff Polynomial Interpolation Algorithm

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Abstract:

Given a list $Z_n = [z_0, \dots, z_n]$ with $(n+1)$ distinct real numbers, let $[\alpha_i^{(0)} < \alpha_i^{(1)} < \dots < \alpha_i^{(p_i)}]$ for $i = 0, 1, \dots, n$, with $p_i \in \mathbb{N}$, be a list of non-negative integers corresponding to the derivation orders considered in point z_i and let $r_{i,k}$, for $i = 0, 1, \dots, n$ and $k = 0, 1, \dots, p_i$, be given real numbers. The Birkhoff polynomial interpolation problem is to search a polynomial P such that P and a number of its derivatives take in these nodes given values; in other words, the Birkhoff polynomial interpolation problem consists in searching for a polynomial P , such that $P^{(\alpha_i^{(k)})}(z_i) = r_{i,k}$, for $i = 0, 1, \dots, n$ and $k = 0, 1, \dots, p_i$. Recently, Rhouni et al. presented a new algorithm for computing the Hermite-Birkhoff interpolation polynomial, called the Recursive Hermite-Birkhoff Polynomial Interpolation Algorithm (RHBPIA). In this work, we will give some properties of the Recursive Hermite-Birkhoff Polynomial Interpolation Algorithm, a new result concerning the explicit formulation of the solution will also be established, the numerical stability result for this algorithm will be obtained independently, and a few examples will be presented.

Keywords: Birkhoff interpolation, Hermite interpolation, Newton's formula, Schur complement, Recursive algorithm, Recursive polynomial interpolation algorithm

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GeoAS-Former: Geometry-Aware Self-Attention for Feature-Free Algorithm Selection on the Traveling Salesman Problem

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Abstract:

The performance of optimization algorithms for the Traveling Salesman Problem (TSP) strongly depends on the structural characteristics of the problem instance, motivating automated algorithm selection approaches that recommend a suitable solver on a per-instance basis. Although recent deep learning methods have reduced the dependence on handcrafted instance features through convolutional, point-cloud, and graph-based representations, effectively capturing both local and global Euclidean structures remains an important challenge. This work proposes GeoAS-Former, a feature-free geometry-aware Transformer for offline algorithm selection on Euclidean TSP instances. Unlike conventional self-attention, GeoAS-Former explicitly incorporates relative Euclidean information into the attention mechanism through a learned geometric bias derived from normalized pairwise distances. The proposed architecture combines geometry-aware attention with local neighborhood and global interactions, enabling the model to capture fine-grained spatial patterns together with instance-level structural dependencies. Since TSP instances represent unordered sets of cities, the architecture avoids sequential positional encoding and produces an instance representation through permutation-invariant aggregation before predicting the most appropriate solver. The proposed framework is evaluated on two complementary algorithm-selection benchmarks: TSP-ISA, containing 1,330 100-city instances with a CLK/LKCC portfolio, and TSP-CNN, containing 1,000 1,000-city instances with an EAX/LKH portfolio. This study investigates whether explicitly embedding Euclidean geometry into self-attention can provide a more effective and structurally informed representation for feature-free TSP algorithm selection.

Keywords: Automated Algorithm Selection, Traveling Salesman Problem, Geometry-Aware Transformer, Self-Attention, Deep Learning, Combinatorial Optimization

Minimization of an Isoperimetric Functional Perturbed by a Regular-Kernel Potential

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Abstract:

In this paper, we investigate the minimization of the shape functional energy defined by the sum of the classical perimeter with a competing potential of a regular kernel, which is maximized by the ball. On the class of measurable subsets of the m -dimensional Euclidean space, which are of finite perimeter and have a fixed volume v , we prove the existence of a minimizer and we determine a condition on the kernel for which the ball becomes the minimizer of the considered problem. We then develop a MATLAB-based numerical algorithm to approximate the shape functional problem for the kernel exponential. The numerical results indicate the existence of a threshold area s_0 , which depends on the area and the regular kernel, such that below which the disk appears to minimize the shape functional among the considered shapes. Also, we augmented the considered shape functional by adding the Fraenkel asymmetry of E multiplied by $2vK_{L1}$, we prove that the ball of volume v is a minimizer of the augmented shape functional.

Oral Session 5. Mathematical Biology

A hybrid mesoscopic/agent-based model for crowd dynamics with emotional contagion

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Abstract:

This work investigates crowd dynamics in evacuation scenarios by incorporating emotional contagion. We develop a hybrid framework that couples a mesoscopic kinetic model for pedestrian motion with an agent-based model of emotional interactions. At the emotional level, symmetric and isotropic influence rules are assumed, so that each individual adapts their emotional intensity to the locally averaged state of their neighbors. Numerical simulations, based on a Monte Carlo method for motion and an agent-based scheme for emotion, examine three representative scenarios: calm crowds without contagion, fully emotional crowds, and mixed populations. The results show that emotional contagion amplifies congestion and increases evacuation times. The proposed dual-scale approach highlights how symmetric emotional interactions can lead to asymmetric and highly unbalanced evacuation patterns, providing new insights for crisis management and safety design.

Keywords: Crowd dynamics, Hybrid modeling, Emotional contagion, Kinetic theory, Agent-based simulation, Emergency evacuation

Dynamics of an SVIHR model with a vaccination-age structure, nonlinear incidence rate and relapse

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Abstract:

This study investigates the global dynamics of an SVIHR model that incorporates vaccination age, a general nonlinear incidence rate, relapse, and infection due to vaccination failure. The model is developed to assess the combined effects of vaccination, hospitalization, and the waning of vaccine-induced immunity on disease prevalence in the presence of relapse. First, the model's well-posedness is established using integrated semigroup theory. We then examine the existence of equilibrium points based on the basic reproduction number R_0 . Second by constructing suitable Lyapunov functionals and applying LaSalle's invariance principle, we demonstrate that the disease-free equilibrium is globally asymptotically stable when $R_0 \leq 1$, and that the endemic equilibrium becomes globally asymptotically stable when $R_0 > 1$. Finally, numerical simulations using the finite difference method validate the theoretical findings and highlight the roles of vaccination, hospitalization, and vaccine failure in controlling and mitigating disease spread.

Keywords: SIVR epidemic model, vaccination age, treatment, generalized nonlinear incidence rate, persistence, Lyapunov functional, global stability

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Dynamic Analysis of a Fractional-Order Epidemic Model with Nonlinear Incidence: Application to COVID-19

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Abstract:

In December 2019, a novel coronavirus emerged in Wuhan, China, and rapidly spread worldwide, leading to the COVID-19 pandemic. In this paper, we investigate a fractional-order epidemic model with a nonlinear saturated incidence rate using the Caputo fractional derivative to describe the dynamics of COVID-19 transmission. The existence, uniqueness, positivity, and boundedness of solutions are established. The disease-free and endemic equilibrium points are determined, and their local stability is investigated using the linearization method. The global stability of the equilibria is analyzed by means of a suitable Lyapunov function. Numerical simulations are performed using MATLAB and the Euler method to illustrate the theoretical results and provide insight into the dynamics of COVID-19 based on real-world epidemiological data.

Keywords: fractional-order epidemic model, nonlinear incidence, COVID-19, Caputo fractional derivative, stability analysis, Lyapunov function

On Fractional-Order Discrete-Time SEIR Model

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Abstract:

This paper proposes a discrete-time SEIR epidemic model formulated using a Caputo-like fractional difference operator, which incorporates memory effects into disease transmission dynamics. Including fractional memory provides a more realistic description of epidemic spread than classical integer-order models. The existence and uniqueness of solutions are established employing the Picard iteration method. The equilibrium points of the system are identified and the basic reproduction number R_0 is derived to characterise disease transmission. The local and global stability of the disease-free and endemic equilibria are analysed in relation to R_0 . Numerical simulations are presented to support the theoretical findings and illustrate the impact of memory effects on epidemic dynamics.

Keywords: fractional-order, discrete-time, SEIR model, Caputo difference operator, memory effects, epidemic dynamics, stability analysis, basic reproduction number

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Analysis of a Diffusive Epidemic Model (SIR) with Crowley–Martin Incidence and Holling Type III Treatment in a Heterogeneous Environment

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Abstract:

We study a reaction–diffusion SIR model that incorporates nonlinear infection and treatment effects. The incidence term is chosen in the Crowley–Martin form

$$f(S, I) = \frac{\alpha(x)SI}{(1 + \beta S)(1 + \gamma I)},$$

and the treatment rate is described by a Holling type III saturating response

$$T(I) = \frac{aI^2}{1 + bI^2}.$$

We prove that the system is well posed by establishing the global existence of positive solutions and deriving uniform bounds. Using variational arguments, we formulate the basic reproduction number R_0 and analyze its threshold properties. In particular, we study the local and global stability of the disease-free equilibrium (DFE). Numerical simulations illustrate and support the analytical results, highlighting how diffusion interacts with nonlinear incidence and saturating treatment in epidemic dynamics.

Keywords: SIR model, Reaction–diffusion equations, Holling type III treatment, Crowley–Martin incidence, Basic reproduction number, Variational methods, Stability analysis

Dynamics and optimal control of a Caputo fractional-order model for poliovirus transmission

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Abstract:

This work investigates the transmission dynamics of poliovirus in the context of oral polio vaccine administration by proposing a fractional-order mathematical model that accounts for both wild-type and vaccine-derived infections. The model is rigorously analyzed to establish the existence, uniqueness, positivity, and boundedness of its solutions, along with the characterization of its equilibrium states. The stability properties of these equilibria are examined in relation to the basic reproduction number R_0 , including the possibility of bifurcation phenomena. A sensitivity analysis is performed to determine the most influential parameters driving the system dynamics. Furthermore, an optimal control framework is developed to assess the impact of intervention strategies on disease spread. Numerical simulations are provided to support the analytical findings and to illustrate the effects of the fractional-order parameter and control measures on the evolution of the epidemic.

Keywords: Poliomyelitis, fractional-order, Liouville-Caputo derivative, basic reproduction number, stability, optimal control

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Global Stability and Treatment Effects in a Fractional-Order Multi-Strain Influenza Model

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Abstract:

This work investigates the dynamics of a multi-strain influenza epidemic through a fractional-order mathematical model incorporating treatment effects. The model describes the interactions between susceptible individuals, individuals infected with influenza strains A, B and C, and the recovered population. The use of fractional derivatives enables the model to account for memory effects in disease transmission. Fundamental mathematical properties of the system, including positivity, boundedness, existence and uniqueness of solutions, are established. The basic reproduction numbers associated with the three strains are derived using the next-generation matrix approach. The global asymptotic stability of the disease-free and strain-specific endemic equilibria is investigated through suitable Lyapunov functions and LaSalle's invariance principle. Numerical simulations illustrate the theoretical results and examine the influence of the fractional derivative order on convergence toward equilibrium states. The simulations also highlight the role of treatment parameters in reducing the prevalence of the different influenza strains. These results demonstrate the relevance of fractional-order models for understanding complex epidemic dynamics and assessing intervention strategies.

Keywords: Fractional-order model, Influenza, Multi-strain epidemic, Global stability, Lyapunov method, Basic reproduction number, Treatment

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Macroscopic modeling and adaptive traffic control urban by conservation equations

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Abstract:

This article proposes a macroscopic approach for modeling and managing urban road traffic, based on the Lighthill–Whitham–Richards (LWR) model. The conservation law is solved numerically using the Godunov scheme. A method is proposed to adapt the duration of the green light according to the traffic density before the intersection. This duration varies between T_{min} and T_{max} . Simulations are used to study queue formation and compare this method with fixed-duration signaling. The results show that the proposed method reduces queue lengths and average waiting times, especially during periods of heavy traffic.

Keywords: Road traffic, Partial differential equations, LWR model, Godunov scheme, Numerical simulation, Conservation law

A mathematical study to inform on optimal strategy involving controlled hospitalization and contact tracing to mitigate the spread of epidemic diseases

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Abstract:

The cost-effective strategies deployed to contain epidemic outbreaks often include a diverse number of public health measures mainly aimed at limiting the spread of diseases and reducing the potential casualties. This mix of interventions is important to sustain admissible levels of control while efficiently managing the cumulative costs that are lost to epidemics. In this work, we present an optimal control study to assess the best strategy combining testing interventions and education campaigns for direct support of detection and hospitalization with contact tracing and monitoring of traced infectees for a swift and effective interruption of transmission chains.

Keywords: Ordinary differential equations, Infectious diseases, Hospitalization, Contact tracing, Optimal control

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A Fuzzy Lotka-Volterra Predator-Prey Model with Uncertain Biological Parameters

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Abstract:

This work presents a Lotka-Volterra-type predator-prey model incorporating fuzzy logic in order to account for the imprecision inherent in biological parameters. We examine the existence, the positivity, as well as the local and global stability of the equilibrium points of the system. The results obtained highlight the impact of uncertainty on the stability of the system. Finally, we discuss new research directions, in particular the extension of this model to fractional derivatives, which would make it possible to incorporate a memory effect into the dynamics of the system.

Keywords: Predator-prey model, Lotka-Volterra system, fuzzy logic, uncertainty, biological parameters, equilibrium points, local and global stability, fractional derivatives, memory effects

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Mathematical Modeling and Analysis of a Fractional-Order Influenza Model with Resistant and Non-Resistant Strains

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Abstract:

Influenza is a common respiratory infectious disease that affects people worldwide. In this paper, we propose a fractional-order epidemic model describing the transmission of two influenza strains: resistant and non-resistant strains. We first prove the existence and uniqueness of a global positive solution. Then, we determine the equilibrium points and calculate the basic reproduction number. The global stability of the equilibrium points is studied using Lyapunov functions and LaSalle's invariance principle. Finally, numerical simulations are carried out to illustrate the theoretical results and the behavior of the model. The results show that the fractional order does not change the stability of the equilibrium points, but it affects the rate of convergence of the solutions toward these equilibria.

Keywords: Fractional order, Influenza, Basic reproduction number, Global stability, multi-strain

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A Mathematical Model for Contact Tracing and Monitoring in Disease Outbreaks with Delay Effects

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Abstract:

In this study, we develop a mathematical model to investigate the effectiveness of contact tracing and monitoring in limiting and containing disease outbreaks. Our modeling framework also accounts for the delays associated with the tracing process and their impact on the efficiency of interventions. To capture the essential of tracing's features above, we adopted a compartmental model jointly with a partial differential equation and coupled state-dependent-boundary conditions. The model is simplified into a system of differential equations with delay which is used to express the effective reproduction number R_e when contact tracing is employed to limit disease spread in terms of the basic reproduction number R_0 , clinically measurable quantities, as well as the delay associated with the tracing process. The obtained formula for R_e is presented in different scenarios to reflect some insights in considering contact tracing as a control measure to mitigate emerging epidemics or highly infectious diseases where clinical/pharmaceutical interventions have not yet been readily accessible or exhibit saturation effect. The model is fitted to data of the Omicron SARS-CoV-2 in South Korea during the 2022 outbreak. Further numerical results are provided to explore the effects of case detection, tracing, delay, and monitoring on R_e .

Keywords: contact tracing, disease outbreaks, compartmental model, partial differential equation, effective reproduction number, SARS-CoV-2, delay differential equations, monitoring

Global Stability of a Dual HIV-1/HIV-2 Model with Drug Resistance under Bilinear Incidence

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Abstract:

This study proposes and analyzes a deterministic compartmental model for the transmission dynamics of HIV-1 and HIV-2 in the presence of drug resistance. Transmission of each strain is governed by a bilinear incidence function, thereby relating the force of infection directly to interactions between susceptible and infected individuals. The population is stratified into susceptible individuals, HIV-1- and HIV-2-infected individuals, a drug-resistant class, an AIDS class, and a removed class. The fundamental properties of the model are first established by proving the positivity and boundedness of solutions, which ensure that the dynamics remain epidemiologically meaningful. Using the next-generation matrix method, the strain-specific basic reproduction numbers $R_{0,1}$ and $R_{0,2}$ are derived. The existence of the disease-free, single-strain endemic, and coexistence equilibria is investigated in relation to these threshold quantities. Local stability is determined through linearization of the system, whereas global asymptotic stability is established by constructing appropriate Lyapunov functions and applying LaSalle's invariance principle. The resulting threshold conditions characterize disease extinction, persistence of a single HIV strain, and coexistence of both strains in the presence of drug resistance. Numerical simulations are used to support the analytical results and to illustrate the asymptotic behavior of the system under different epidemiological regimes. The analysis provides a rigorous mathematical framework for describing the long-term dynamics of dual HIV-1/HIV-2 transmission and the associated emergence of drug resistance under bilinear incidence.

Keywords: HIV-1, HIV-2, drug resistance, bilinear incidence, basic reproduction number, coexistence equilibrium, global stability, Lyapunov function

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The Global Dynamics of the Fractional SEIQR Epidemic

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Abstract:

This paper investigates the global dynamics of a fractional-order epidemic model with five compartments: susceptible, exposed, infectious, quarantined, and recovered individuals. The temporal evolution incorporates memory effects through a fractional derivative [1, 2, 3], while spatial movement is modeled by diffusion with homogeneous Neumann boundary conditions. We establish the well-posedness of the system by proving existence, uniqueness, positivity, and boundedness of solutions. A threshold parameter, the basic reproduction number R_0 , is derived and shown to completely determine the asymptotic behavior: when $R_0 \leq 1$ the disease-free equilibrium is globally asymptotically stable, whereas for $R_0 > 1$, a unique endemic equilibrium emerges and attracts all positive solutions. The analysis further reveals that fractional memory suppresses spatial pattern formation, leading to more homogeneous disease distribution [4, 5]. Numerical simulations using an L1-finite difference scheme validate the theoretical findings and demonstrate that decreasing the fractional order attenuates epidemic peaks. The study concludes with perspectives on extending the framework to abstract Hilbert spaces for a more general functional analytic treatment.

Keywords: Fractional differential equations, SEIQR model, Reaction-diffusion

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A Caputo Fractional-Order Model for the Social Diffusion of Physical Inactivity and Obesity

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Abstract:

Background. Physical inactivity is one of the leading modifiable risk factors for non-communicable disease. Longitudinal social-network studies show that sedentary behaviour and weight gain do not merely co-occur within individuals: they diffuse along social ties through peer reinforcement, a mechanism whose functional form is that of a force of infection. **Model.** We propose and analyse a five-compartment model — active susceptibles, physically inactive individuals, obese individuals, individuals under lifestyle intervention, and individuals who have returned to an active lifestyle — in which the onset of inactivity is driven by a nonlinear force of social exposure, and in which memory is incorporated through the Caputo fractional derivative of order $\alpha \in (0, 1]$, so that current dynamics depend on the whole history of past exposure rather than on its instantaneous level alone. The model further includes a relapse loop returning part of the reactivated population to the inactive class. **Mathematical results.** We establish well-posedness: non-negativity and uniform boundedness of solutions in a biologically feasible invariant region, and existence and uniqueness via a Picard–Banach fixed-point argument under an explicit contraction condition. The basic reproduction number R_0 is derived by the next-generation matrix method and displays explicitly the amplifying effect of the relapse loop. The inactivity-free equilibrium is locally asymptotically stable for $R_0 < 1$, the fractional statement following from Matignon’s criterion, and its global stability is proved along two independent routes: a Lyapunov functional combined with the fractional LaSalle invariance principle, and a Metzler linear comparison system. Beyond the threshold, a centre-manifold (Castillo-Chavez–Song) analysis shows that the transcritical bifurcation at $R_0 = 1$ is forward, ruling out sub-threshold bistability and hysteresis; the endemic equilibrium is constructed explicitly and shown to be unique and Matignon-stable for every $\alpha \in (0, 1]$. **Numerical results and interpretation.** A normalized forward sensitivity analysis identifies the peer-influence coefficient, of unit elasticity, followed by the relapse rate, as the dominant drivers of R_0 . Simulations performed with the fractional Adams–Bashforth–Moulton predictor–corrector scheme, whose convergence order is verified against an exact Mittag-Leffler solution, show that smaller values of α delay the inactivity peak while sustaining a higher long-run burden. Since the equilibria are independent of α , memory governs the speed of the dynamics rather than its limiting state, thereby reinforcing the case for early intervention.

Keywords: fractional-order model, Caputo derivative, social contagion, physical inactivity and obesity, basic reproduction number, Matignon criterion, forward bifurcation, sensitivity analysis

A SIQR Epidemic Model with Contact Tracing: From Discrete to Continuous Time, Infection-Age Structure, Susceptibility Heterogeneity, and Optimal Control

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Abstract:

We develop a continuous SIQR (Susceptible–Infected–Quarantined–Recovered) epidemic model that incorporates contact tracing explicitly, a mathematical discrete-time compartmental model is used. Under the condition of effective reproduction number $R_e < 1$, the disease-free equilibrium is locally asymptotically stable. Then to structure the population infected, an infection age is considered as linear chain composed of n stages with infectiousness β_k and detection δ_k . We further formalize this limit as a genuine age-structured transport PDE with contact tracing embedded either as a local, capacity-constrained removal rate $\delta(a, t) = \delta_{max}(a)W/(W + cB(t))$ or as a nonlocal, mechanistically derived backward-reconstruction term (following Ladib, Ouhinou & Yakubu, 2024), and cast the resulting system as an abstract semilinear Cauchy problem with nonlinear boundary conditions on a Banach space Webb (1985) and Magal & Ruan (2018). Finally, we compare our framework to the mechanistic contact-tracing model of Ladib, Ouhinou & Yakubu (2024), adopt their key insight of deriving the detection rate endogenously from a backward-reconstruction mechanism rather than imposing it exogenously, and propose a machine-learning-based, data-driven framework.

Keywords: epidemic modeling, contact tracing, SIQR model, semilinear Cauchy problem, stability analysis, infection-age structure, McKendrick–von Foerster equation, herd immunity threshold, susceptibility heterogeneity, optimal control, disease surveillance capacity

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Mathematical Modeling and Optimal Control Analysis of Melioidosis Transmission Dynamics

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Abstract:

Melioidosis is an infectious disease caused by the bacterium *Burkholderia pseudomallei*, which poses a significant public health threat in tropical and subtropical regions. In this work, we developed and analyzed a mathematical model of the melioidosis epidemic in order to gain insight into the mechanisms driving the spread of the disease within a population. The basic reproduction number, R_0 , was derived using the next-generation matrix approach, and its sensitivity to the key epidemiological parameters was assessed numerically. An optimal control problem was then formulated and analyzed using Pontryagin's maximum principle to derive the optimality system, and the results highlight the relative importance of the different parameters in shaping the progression of the epidemic and provide a better understanding of the factors that govern the persistence and spread of melioidosis. These findings may serve as a useful theoretical basis for informing public health interventions and future research on the control of melioidosis.

Keywords: Melioidosis, *Burkholderia pseudomallei*, Sensitivity Analysis, Optimal Control, Pontryagin's Maximum Principle

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Maximal Regularity and a Spectral PINN Method for Fractional Equations with Memory

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Abstract:

We study a class of periodic time-fractional integro-differential equations with Abel-type infinite memory of the form

$$D_{CW}^\alpha u(t) + Au(t) + AI^\beta u(t) = f(t), \quad t \in \mathbb{T},$$

where A is a closed linear operator on a UMD Banach space X , D_{CW}^α denotes the periodic Caputo–Weyl fractional derivative of order $\alpha \in (0, 1)$, and I^β is the Abel-type memory operator of order $\beta \in (0, 1)$. We characterize L^p -maximal regularity of this equation in terms of the $\mathcal{R}$ -boundedness of an explicit family of operator-valued Fourier multipliers built from the symbol $(ik)^\alpha + A(1 + (ik)^{-\beta})$. The result extends the framework of Bu–Fang, Keyantuo–Lizama, and Bahloul to combined fractional differentiation and Abel memory operators. As a numerical counterpart, we develop a *spectral physics-informed neural* method in which the fractional derivative and the memory operator are evaluated through their exact Fourier symbols, thereby avoiding direct discretization of singular convolution kernels. Convergence of the discrete scheme is analyzed under the maximal regularity assumption and quantified in periodic Besov spaces. Numerical experiments on a scalar prototype, on a fractional-in-time anomalous diffusion model, and on a viscoelastic creep identification problem confirm the theoretical predictions and illustrate the robustness of the method under noisy and sparse data.

Oral Session 6. Fractional Calculus and Fractional Differential Equations

Existence and uniqueness of solutions to nonlocal value problems for fractional integro-differential evolution systems involving a ϕ -Caputo derivative

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Abstract:

This research focuses on a detailed investigation of the existence and uniqueness of solutions to nonlocal value problems for fractional integro-differential evolution systems involving a ϕ -Caputo derivative. Specifically, we derive the solution formula in terms of the semigroup generated by the resolvent and the ϕ -function associated with the Caputo fractional derivative, using the generalized Laplace transform method with the aid of certain probability density functions. To establish the existence of solutions, we utilize Mönch's fixed point theorem, while Banach's fixed point theorem is employed to confirm uniqueness. Finally, we illustrate the main results and their implications within the context of semigroups and fractional derivatives using two concrete examples

Keywords: fractional integro-differential evolution systems, ϕ -Caputo derivative, nonlocal value problems, Mönch's fixed point theorem, Banach's fixed point theorem, generalized Laplace transform, semigroup

Existence of Mild Solutions for a Time-Fractional Burgers' Equation

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Abstract:

This work addresses a fractional version of the one-dimensional viscous Burgers' equation with a Caputo time derivative of order $0 < \alpha < 1$. The problem is posed on $(0, 1)$ with homogeneous Dirichlet boundary conditions and initial data in $L^2(0, 1)$.

We consider the following fractional Burgers' equation:

$$\begin{cases} \partial_t^\alpha w(x, t) = \mu \partial_{xx} w(x, t) - w(x, t) \partial_x w(x, t), & x \in (0, 1), \quad t \in [0, T], \\ w(0, t) = w(1, t) = 0, & t \in [0, T], \\ w(x, 0) = w_0(x), & x \in (0, 1). \end{cases}$$

where ∂_t^α is the Caputo fractional derivative of order $0 < \alpha < 1$, $T > 0$, and $\mu > 0$ is the viscosity coefficient.

By reformulating the equation as a nonlinear integral problem involving a fractional resolvent operator, we establish the existence of a local-in-time mild solution in $C([0, T], L^2(0, 1))$. The proof is based on compactness arguments and Schauder's fixed point theorem.

Keywords: Burgers' equation, Caputo fractional derivative, mild solution, memory effects, Schauder fixed point theorem

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Analytical and Numerical Study of a Fractional Burgers Equation with Mixed Derivatives

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Abstract:

A nonlinear fractional Burgers equation involving an Atangana–Baleanu fractional derivative in time and a Caputo fractional derivative in space is considered. The model can be regarded as a fractional extension of the classical Burgers equation, through which memory and spatial nonlocality may be incorporated into the mathematical description. Particular attention is given to the well-posedness of the resulting problem, with emphasis on the existence, uniqueness, and convergence of solutions. An analytical procedure combining the variational iteration method with the Laplace transform is employed, and appropriate conditions are established under which the resulting solution can be obtained and its convergence analyzed. Numerical simulations are subsequently carried out to assess the applicability of the proposed approach and to examine the influence of the fractional orders on the solution profiles. The obtained results indicate that the fractional orders may significantly affect the temporal and spatial behavior of the solutions. These findings suggest that the proposed fractional formulation can provide a suitable mathematical framework for describing Burgers-type phenomena in which memory and nonlocal effects are relevant.

Keywords: Fractional Burgers equation, Atangana–Baleanu fractional derivative, Caputo fractional derivative, well-posedness, existence and uniqueness, convergence, variational iteration method, Laplace transform, numerical simulation

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Semigroups and Cosine Operators for Hybrid Fractional ψ -Caputo Equations of Order $\alpha \in (1, 2)$

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Abstract:

In this communication, we prove the existence of mild solutions for a hybrid fractional differential equation with ψ -Caputo operators of order $1 < \alpha \leq 2$, under combined Lipschitz and Carathéodory conditions. The solutions are constructed using the generalized ψ -Laplace transform and new density functions. The proofs are based on the fixed point theorem and key methods from ψ -fractional calculus.

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On Nonlinear ψ -Hilfer Fractional Elliptic Problems with Variable Exponents and Singular Terms: Existence and Regularity

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Abstract:

We consider a nonlinear elliptic problem involving a singular nonlinearity, lower-order terms, and L^1 -data within the framework of ψ -Hilfer fractional Sobolev spaces with variable exponents on a bounded interval. The problem is formulated in terms of the ψ -Hilfer fractional derivative. We prove the existence of weak positive solutions and obtain improved regularity properties. In particular, we show that the presence of the lower-order terms produces a regularizing effect, allowing the solutions to gain additional Sobolev regularity beyond what can be expected from the singular nonlinearity and the low integrability of the data. The proof is based on a combination of variational arguments, truncation techniques, and compactness methods adapted to the variable-exponent fractional setting.

Keywords: ψ -Hilfer fractional derivative, fractional Sobolev spaces, variable exponent, singular nonlinearity, weak positive solutions, regularity

Controllability of mild solutions for an abstract fractional Cauchy problem

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Abstract:

This paper is concerned with the controllability of mild solutions for an abstract fractional Cauchy problem. The approach is based on fixed-point methods and the theory of families of linear operators.

Keywords: Mild solution, controllability, fractional derivatives, families of linear operators, fixed point

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Caputo Sweeping Process over Convex Sets and Simulations

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Abstract:

J. J. Moreau first formulated the concept of the "sweeping process" in 1971 to model elastic-plastic mechanical systems. This abstract presents a specific class of sweeping processes characterized by a fractional ψ -Caputo derivative in Hilbert spaces, which is expressed by the inclusion:

$$-{}_0^c D_t^{\alpha, \psi} x(t) \in \mathbb{N}_{\kappa(t)}(A({}_0^c D_t^{\alpha, \psi} x(t)) + Bx(t))$$

We prove the uniqueness of the solution of sweeping processes through Rothe's method. Moreover, the established abstract results are applied to investigate a complex viscoelastic contact problem involving fractional constitutive laws. These theoretical results are supported by numerical simulations.

Keywords: Caputo fractional, Sweeping Process, Existence and Uniqueness, Contact Problem

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An Adaptive Multi-Index Fractional Power Series Method for Solving Fractional Differential Equations in the Caputo Sense

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Abstract:

In this work, we introduce a new general form of the fractional power series method for solving non-trivial fractional differential equations in the Caputo sense. In contrast to classical fractional power series, the proposed expansion adaptively adjusts the number of indices appearing in its exponents according to the specific structure of the equation under consideration. This flexibility widens the class of fractional differential equations that can be treated within a single unified framework. Several examples are provided to illustrate the applicability and the effectiveness of the proposed approach.

Keywords: Fractional ordinary differential equations, Power series, Caputo derivative, Special functions

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Fractional Memory Effects on the Controllability of Complex Network Systems in Banach Spaces

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Abstract:

The present work is devoted to the mathematical analysis of controllability for nonlinear fractional complex networks in Banach spaces. The underlying dynamics are described by means of a generalized φ -Caputo fractional derivative, which provides a flexible representation of nonlocal memory effects through a strictly increasing temporal function φ . Sufficient conditions for the controllability of linear and nonlinear network systems are established using semigroup theory and fixed-point methods. The investigation further examines the influence of network topology and temporal scaling on the attainable states and the controllability properties of the system. Numerical simulations are conducted to illustrate the theoretical results and assess the impact of fractional memory on control behavior. This analysis contributes to the development of a rigorous functional framework for the controllability study of complex network dynamics governed by fractional operators.

New model using the fractional time derivatives for image denoising

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Abstract:

We present a novel image denoising model involving fractional time derivatives for capturing image texture, and an efficient regularization to reduce the staircase effect. We show the well-posedness of a weak solution and illustrate the numerical results obtained in the context of image denoising.

Well-posedness for an improved time-fractional reaction-diffusion systems

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Abstract:

This work reports on a time-fractional reaction-diffusion system for image denoising. Inspired by the H^{-1} image decomposition strategy, the model couples a restored image u with an auxiliary variable v . It combines a Weickert-type anisotropic diffusion tensor with a fractional Laplacian and Caputo time derivatives, incorporating both spatial nonlocality and temporal memory. For a noisy image f , the system reads

$$\begin{cases} {}^C D_t^\alpha u - \operatorname{div}(D(J_\rho(\nabla u_\sigma))\nabla u) + \lambda v = 0, \\ {}^C D_t^\alpha v + \beta(-\Delta)^s v + f - u = 0, \end{cases} \quad 0 < \alpha, s < 1,$$

with homogeneous local and nonlocal Neumann conditions and initial data $u(0) = f$, $v(0) = 0$. Here u_σ is a Gaussian-smoothed image, J_ρ is its structure tensor, and $\lambda, \beta > 0$.

Under the stated smoothness and coercivity assumptions on the diffusion tensor and for $f \in L^2(\Omega)$, we establish existence and uniqueness of a weak solution, with $u \in L^2(0, T; H^1(\Omega))$ and $v \in L^2(0, T; W^{s,2}(\Omega))$. The analysis uses fractional energy estimates, compactness, the Schauder fixed-point theorem, and a fractional Gronwall inequality. A finite difference discretization is then used to evaluate the model on synthetic, natural, and medical images. Experiments with Gaussian and Poisson noise show improved peak signal-to-noise ratio and structural similarity relative to the tested PDE-based methods, together with effective edge and texture preservation. These results support the use of temporal memory and nonlocal diffusion in image restoration.

Oral Session 7. Inverse Problems, Imaging and Regularization

Image Deblurring via Regularization by Denoising and L^1/L^2 Gradient-Ratio Regularization

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Abstract:

Image deblurring remains a fundamental problem in image processing. We propose a new variational model combining regularization by denoising (RED) with the L^1/L^2 gradient ratio as a novel regularization term, jointly balancing data fidelity and image smoothness. The resulting minimization problem is solved via an ADMM-based scheme with convergence guarantees. Experiments show the proposed model outperforms several competing approaches.

Keywords: Image deblurring, regularization by denoising, L1/L2 gradient

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A Learnable Descent Method for Estimating the Source Term in a Subdiffusion Model

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Abstract:

Fractional diffusion equations provide a powerful framework for modeling anomalous diffusion phenomena that classical models fail to capture. In this work, we investigate the inverse problem of recovering an unknown space-dependent source term in a time-fractional (subdiffusion) equation from noisy final-time measurements. The identification problem is formulated as a nonsmooth, nonconvex optimal control problem, in which the regularization term is not fixed a priori but is instead learned through a deep neural network, following the Learnable Optimization Algorithm paradigm. To solve this nonsmooth and nonconvex minimization problem, we introduce a Learnable Descent Algorithm (LDA) that combines a Nesterov-smoothed proximal gradient step with a safeguard gradient-descent step, selecting at each iteration the candidate that yields the lowest value of the objective functional. On the theoretical side, we establish the well-posedness of the direct and adjoint problems, derive the Lipschitz continuity of the control-to-state and control-to-adjoint-state mappings, obtain a closed-form expression for the gradient of the fidelity term via the adjoint equation, and prove, using Clarke's subdifferential calculus, that every accumulation point generated by the algorithm is a Clarke stationary point of the objective functional. Extensive numerical experiments in one and two space dimensions, including smooth, piecewise-linear, discontinuous, and sawtooth source terms, under noise levels ranging up to 70%, demonstrate that the proposed LDA consistently outperforms the classical Alternating Direction Method of Multipliers (ADMM) in terms of reconstruction accuracy and robustness to noise, while remaining effective for irregular and singular source profiles.

Keywords: Inverse problem, Optimal control problem, Learnable Descent Algorithm (LDA), Fractional partial differential equations (FPDEs)

Effective shadow mitigation in image processing using time fractional anisotropic osmosis with bilateral total variation

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Abstract:

We propose a time-fractional anisotropic osmosis model for image shadow removal, incorporating bilateral total variation (BTV) regularization to preserve edges and structural details. The model employs a Caputo fractional derivative to introduce temporal nonlocality into the osmosis evolution while effectively correcting illumination inconsistencies. The analytical properties of the resulting fractional PDE are investigated, and a finite-difference scheme is developed for its numerical approximation. The proposed discretization is designed to preserve the main structural properties of the continuous model and ensure numerical stability under suitable conditions. Numerical experiments demonstrate the effectiveness of the approach in reducing shadows while maintaining significant image features. Comparisons with classical osmosis and diffusion-based methods further illustrate the potential of combining fractional-order dynamics with BTV regularization for robust shadow removal.

Keywords: Fractional time derivative, Caputo derivative, anisotropic osmosis equation, nonlinear diffusion, bilateral total variation, shadow removal, image processing

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Shape optimization with uncertain boundary conditions in non-coercive PDEs

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Abstract:

This work addresses a shape optimization problem governed by a non-coercive partial differential equation, with uncertainty affecting the boundary condition on the measurement boundary. A variational framework is considered to account for the influence of boundary uncertainties on the optimal design. The study focuses on the impact of these uncertainties on the behavior of the solution as well as on the resulting optimal configurations.

Keywords: Shape optimization, Non-coercive PDE, Neumann boundary condition, Optimal design, Boundary uncertainty

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Cavity shape reconstruction with a homogeneous Robin condition via a constrained coupled complex boundary method with ADMM

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Abstract:

We revisit the problem of identifying an unknown portion of a boundary subject to a Robin condition, based on a pair of Cauchy data on the accessible part of the boundary. It is known that a single measurement may correspond to infinitely many admissible domains. Nonetheless, numerical strategies based on shape optimization have been shown to yield reasonable reconstructions of the unknown boundary. In this study, we propose a new application of the coupled complex boundary method to address this class of inverse boundary identification problems. The overdetermined problem is reformulated as a complex boundary value problem with a complex Robin condition that couples the Cauchy data on the accessible boundary. The reconstruction is achieved by minimizing a cost functional constructed from the imaginary part of the complex-valued solution. To improve stability with respect to noisy data and initialization, we augment the formulation with inequality constraints through prior admissible bounds on the state, leading to a constrained shape optimization problem. The shape derivative of the complex state and the corresponding shape gradient of the cost functional are derived, and the resulting problem is solved using an alternating direction method of multipliers (ADMM) framework. The proposed approach is implemented using the finite element method and validated through various numerical experiments.

Keywords: Inverse boundary identification, Robin boundary condition, Shape optimization, CCBM (coupled complex boundary method), ADMM (Alternating Direction Method of Multipliers)

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A Coupled Complex Boundary Method for Inverse Conductivity Problems with Unknown Inclusion

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Abstract:

In this work, we present and investigate a novel strategy for the reconstruction of an unknown perfectly conducting inclusion in an inverse conductivity problem. More precisely, the aim is to determine the location and shape of an inclusion ω contained within a bounded domain Ω , based solely on measurements prescribed on the outer boundary $\partial\Omega$. Our approach relies on the Coupled Complex Boundary Method (CCBM), which transforms the original inverse problem into a complex-valued boundary value problem. The resulting formulation incorporates a complex Robin-type boundary condition that establishes a coupling between the Dirichlet and Neumann boundary data. The unknown inclusion is subsequently reconstructed through the minimization of a shape-dependent cost functional. This functional is constructed using the imaginary component of the complex-valued solution throughout the computational domain, providing an effective framework for identifying the geometry of the perfectly conducting obstacle.

Structure-Preserving Shadow Removal via Fractional Graph-Biharmonic Flow Matching

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Abstract:

Shadow removal requires correcting illumination in shadowed regions while preserving the structural and photometric information of unaffected areas. We propose a fractional graph-biharmonic conditional flow-matching framework for structure-preserving shadow removal. The method estimates a masked logarithmic illumination correction that vanishes outside the shadow mask, ensuring exact preservation of non-shadow pixels. A weighted nonlocal graph connects shadowed regions with distant illuminated regions having similar illumination-invariant appearance. A zero-exterior biharmonic operator and its fractional spectral power are then used to construct a graph-aware transport path for conditional flow matching, promoting long-range illumination propagation while limiting undesired texture and reflectance modifications. The predicted correction is further decomposed into a graph-scale illumination component and a local penumbra residual to recover both global illumination variations and fine shadow-boundary transitions. Theoretical results establish coercivity, stability with respect to graph and forcing perturbations, and consistency of the flow-matching formulation. Experiments over three datasets shows an improved shadow restoration while preserving regions outside the mask exactly. Ablation studies further demonstrate the contributions of graph construction, fractional operator design, flow matching, and mask information.

Keywords: Shadow removal, Conditional flow matching, Fractional graph operators, Graph biharmonic operator, Image restoration, Structure preservation

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Fractional Reaction–Diffusion Models for Image Enhancement: Recent Approaches and a New Model

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Abstract:

Fractional reaction–diffusion models have attracted increasing attention in image processing due to their ability to describe nonlocal effects and provide greater flexibility than classical integer-order diffusion equations. Several fractional reaction–diffusion approaches for image enhancement are considered, with particular emphasis on their mathematical formulation and their ability to preserve significant image structures while reducing noise. Different fractional operators and nonlinear diffusion mechanisms are discussed, highlighting the role of memory effects and anomalous diffusion in improving the balance between smoothing and edge preservation. A new fractional reaction–diffusion model for image enhancement is then introduced, combining fractional dynamics with a nonlinear diffusion mechanism adapted to local image features. The mathematical properties of the proposed model are investigated, together with an appropriate numerical approximation. Numerical experiments on representative test images illustrate the effectiveness of the proposed approach in reducing noise, preserving edges and relevant structures, and improving the visual quality of the restored images. The obtained results confirm that fractional reaction–diffusion modeling provides a flexible and promising framework for advanced image enhancement.

Keywords: Fractional calculus, Image denoising, Caputo derivative, Variational methods, Finite difference scheme

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A Deep Learning Framework for Shadow Removal Driven by Anisotropic PDEs

Fakhr Limami, Amine Laghrib, Aissam Hadri, Lekbir Afraites

Abstract:

We propose a physics-informed deep learning model for image shadow removal leveraging spatially varying anisotropic PDEs and tensor operators. The underlying continuous diffusion model is discretized via a nine-point stencil finite-difference scheme—decomposing 2D spatial diffusion into four directional 1D paths—and mapped directly onto the residual blocks of a ResNet. Configured end-to-end, the framework performs shadow detection to generate a binary mask, guiding the PDE-driven ResNet sub-network for precise shadow removal.

Keywords: Shadow removal, PDEs, Finite difference methods, Neural networks

Langevin-within-Split Gibbs Sampling for Bayesian Image Restoration with Learned Group-Sparsity Priors

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Abstract:

****Abstract****

Bayesian image restoration requires effective prior models and computationally efficient sampling strategies to quantify uncertainty in ill-posed inverse problems. In this work, we propose a Langevin-within-Split Gibbs (LWSG) sampler for Bayesian inverse problems involving a learned group-sparsity regularizer. The regularization functional is constructed from nonlinear learned features and smoothed using a Nesterov–Huber approximation, yielding a differentiable energy suitable for gradient-based sampling.

The proposed splitting strategy introduces an auxiliary variable to decouple the data-fidelity term from the learned prior. For linear inverse problems with Gaussian noise, the image variable can be sampled exactly from a Gaussian conditional distribution, while the auxiliary variable is updated using a single unadjusted Langevin step. This combination exploits the structure of the observation model while retaining the flexibility of learned regularization.

We establish sufficient conditions for the propriety of the prior and posterior distributions and analyze the stability of the resulting Markov chain. In particular, we derive explicit conditions under which the quadratic splitting term compensates for the weak nonconvexity of the learned regularizer, leading to geometric convergence in Wasserstein distance. We further provide a quantitative bound on the discretization bias induced by the Langevin step. The theoretical guarantees are restricted to the stated contraction regime and do not directly cover rank-deficient forward operators.

Numerical experiments on image deblurring and super-resolution demonstrate the potential of the proposed method for image reconstruction and uncertainty quantification. Comparisons with variational and sampling-based methods assess reconstruction quality, sampling efficiency, and posterior uncertainty, including pixelwise standard-deviation maps and credible-interval coverage.

****Keywords:**** Bayesian inverse probl

DN-PINNs: A Dirichlet–Neumann Physics-Informed Neural Network Iteration for Inverse Cauchy Problems

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Abstract:

We consider the elliptic Cauchy problem of recovering the missing trace and conormal flux on an inaccessible boundary Γ_0 from overspecified data on an accessible part Γ_2 for $-\nabla \cdot (\xi \nabla u) = f$ in $\mathcal{D}$, with $u = u_d$ and $\xi \nabla u \cdot n = g_2$ on Γ_2 . We study DN-PINNs, a single-domain neural realization of the complementary Kozlov–Maz’ya–Fomin (KMF) iteration [1]. Two physics-informed neural networks sequentially solve well-posed mixed Dirichlet and Neumann problems and exchange trace and conormal flux on Γ_0 ; the trace is updated by $\phi^{(k+1)} = (1 - \omega)\phi^{(k)} + \omega u_N^{(k)}|_{\Gamma_0}$. For a rectangular benchmark, the exact sine-mode factor is $\rho_{\omega,m} = 1 - \omega \operatorname{sech}^2(m\pi)$. Every fixed finite modal space is contractive, but no contraction factor strictly below one is uniform over all modes; the same-type pairing is expansive. A controlled finite-element experiment reproduces the contrasting error trends of the two pairings. The neural method is analysed as an inexact fixed-point iteration under explicit stable-subspace and continuous-residual assumptions; a small empirical collocation loss alone does not provide the required Sobolev residual control. The iterative-regularization perspective is relevant for ill-posed inverse problems [3]. Related PINN-assisted relaxed alternating methods have been studied for image inpainting [4]; the present work focuses on elliptic Cauchy reconstruction. With fixed relaxation $\omega = 0.85$, the clean 500-iteration DN-PINN experiment attains a post-hoc sampled minimum of $E_D = 0.0310$ at $k = 428$, followed by $E_D = 0.0999$ at $k = 500$. Among nine noisy runs at relative levels 5%, 10%, and 20%, eight attain an interior sampled minimum and one shows no turnover within $k \leq 500$. The accessible-discrepancy threshold is crossed in five runs and does not generally coincide with the truth-based sampled minimum. These results indicate realization-dependent behavior, without establishing seed-independent noise robustness or a universally reliable stopping rule. Exact inaccessible data are used only for retrospective evaluation.

Keywords: Inverse Cauchy problem, Kozlov–Maz’ya–Fomin iteration, physics-informed neural networks, iterative regularization, semi-convergence

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A Comparative Study of Classical Optimization Methods for Total Variation-Based Image Denoising

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Abstract:

Image denoising is a fundamental inverse problem in image processing that requires recovering meaningful image structures from noisy observations while preserving edges and fine details. A variational approach to this problem using Total Variation (TV) as a regularizer provides a theoretical framework for solving this problem. However, the non-differentiability of the TV functional presents computational challenges that require appropriate numerical optimization techniques. In this work, we present a comparative study of two classical optimization approaches for TV-based image denoising: the Primal-Dual method and the Alternating Direction Method of Multipliers (ADMM), together with their accelerated variants. The Primal-Dual approach relies on a saddle-point formulation involving primal and dual variables, whereas ADMM employs variable splitting and augmented Lagrangian techniques to handle the non-smooth regularization term. Numerical experiments are conducted under additive Gaussian noise to evaluate the different approaches in terms of reconstruction quality, convergence behavior, and computational cost. This comparative study highlights the mathematical and computational characteristics of these optimization techniques and provides insights into their efficiency for variational image restoration.

Oral Session 8. Dynamical Systems, Mathematical Physics and Control

Exponential Stabilization of Infinite-Dimensional Bilinear Systems by Boundary Controls

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Abstract:

The main objective of this chapter is to investigate the exponential stabilization of infinite-dimensional bilinear systems evolving on a spatial domain $\mathcal{O} \subset \mathbb{R}^n, n = 1, 2, 3$. The system is driven by a bilinear control acting on the boundary $\partial\mathcal{O}$. Under suitable observability conditions, a boundary feedback control is designed to ensure exponential stabilization. Numerical examples and simulations are provided to illustrate the effectiveness of the proposed approach.

Keywords: Infinite-dimensional bilinear systems, exponential stabilization, boundary control, observability

(α, ϕ) -conformable integro-differential equation with nonlocal condition using new fractional conformable derivative and C-semigroup.

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Abstract:

In this work, we introduce the notion of fractional conformable (α, ϕ) -C-semigroup and its infinitesimal generator, and we investigate some of their common properties. Then, and as an application, we study an (α, ϕ) -conformable integro-differential equation with nonlocal condition defined via (α, ϕ) -C-semigroup and under conformable (α, ϕ) -derivative. Furthermore, we prove the existence, the uniqueness of the mild solution and the continuous dependence between mild solutions and initial data for the foregoing Cauchy problem. Finally, we study the asymptotic (α, ϕ) -stability for this kind of solutions.

Keywords: (α, ϕ) -conformable integro-differential equation, nonlocal condition, fractional conformable derivative, C-semigroup, mild solution, asymptotic (α, ϕ) -stability

Optics and Thermodynamics of Charged BTZ Black Holes with Exotic Matter Sources

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Abstract:

We investigate the optical and thermodynamic properties of charged BTZ black holes surrounded by exotic matter sources, including a quintessence-like anisotropic fluid and a string cloud. The resulting geometry incorporates the effects of both the electromagnetic field and the surrounding matter distribution, leading to modified horizon and photon-orbit structures. We analyze the corresponding black-hole thermodynamics and study the conditions under which stable photon circular orbits can arise. In particular, we examine how the parameters associated with the exotic matter sources modify the critical radii, thermodynamic quantities, and optical properties of the charged BTZ geometry. We further discuss the implications of these modifications for the consistency and stability of the black-hole configurations. The results provide a framework for exploring how non-standard matter sources affect the interplay between black-hole optics and thermodynamics in lower-dimensional gravity.

Keywords: BTZ black holes, optics, thermodynamics, exotic matter, quintessence, string cloud, photon orbits, lower-dimensional gravity

Inverse Optimal Control and Robust Stabilization of Infinite-Dimensional Semilinear Systems

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Abstract:

This paper presents an inverse optimal control problem for infinite-dimensional semilinear systems. We are looking for optimality conditions such that the control law obtained by the standard linear quadratic problem of the linear part is optimal for the semilinear system associated to an adequate functional cost. The test of stability with respect to a bounded perturbation in openloop dynamics is assured in terms of the gain and phase margins, once the semilinear closed-loop system's optimality is established. Furthermore, we give conditions for which the trajectories of the closed-loop semilinear system converge exponentially towards its equilibrium. Finally, some numerical results illustrate the effectiveness of our approach.

Keywords: Linear quadratic problem, semilinear system, optimal control, stabilisation, robustness

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Multi-Parameter Bifurcation Analysis and Hybrid Control of a Discrete Prey–Predator System with Prey Harvesting

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Abstract:

We investigate a discrete-time predator–prey model with density-dependent prey harvesting. Through analytical and numerical methods, we establish positivity conditions for biologically feasible solutions, characterize the existence and stability of equilibria in (G, E, α) -space, identify codimension-1 and codimension-2 bifurcations via multi-parameter analysis, and explore complex dynamics through bifurcation diagrams and phase portraits. Our results reveal that increasing harvesting effort E can induce stability-loss bifurcations, such as period-doubling and Neimark–Sacker types, leading to chaotic dynamics. Critical thresholds for sustainable harvesting are identified, highlighting conditions under which population collapse may occur. Furthermore, a hybrid control strategy is proposed to suppress the chaotic behavior and the bifurcations induced by harvesting, thereby stabilizing the system at a positive equilibrium. Numerical simulations confirm the effectiveness of the proposed control in restoring stable coexistence of both species.

Keywords: Bifurcation, prey harvesting, difference equations, chaos

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Stability and Periodic Intervention in a Discrete-Time Dynamical Model with Application to Artificial Intelligence Adoption

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Abstract:

This paper develops and analyzes a discrete-time nonlinear dynamical system for modeling artificial intelligence (AI) adoption under government intervention. The model consists of four interacting state variables representing enterprises outside the active adoption process, unsupported AI adopters, government-supported AI adopters, and aggregate AI-related market pressure. The fundamental mathematical properties of the system are first established, including non-negativity and boundedness of solutions. A threshold quantity governing the emergence of AI adoption is then derived, and sufficient conditions for the local asymptotic stability of the AI-free equilibrium are obtained. A sensitivity analysis is carried out to quantify the influence of the principal model parameters on the threshold dynamics. Particular attention is devoted to periodic government intervention. By formulating the resulting periodically varying discrete system and analyzing its monodromy matrix, a spectral stability criterion is established and a critical intervention period is identified. Numerical simulations complement the theoretical analysis and illustrate the transition between stable and persistent adoption regimes under different intervention intensities and frequencies. The results provide a discrete-time mathematical framework for analyzing threshold behavior, stability, and periodic control mechanisms, with AI adoption serving as a contemporary application.

Keywords: Artificial Intelligence adoption, Discrete-time dynamical system, Threshold dynamics, Stability analysis

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Financial Modeling with Variable-Order Fractional Networks: Coupled Agents, Time-Varying Memory, and Endogenous Policy Feedback

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Abstract:

In this work, we develop a general financial modeling framework in which market confidence, capital flows, and policy signals evolve together on a network of interacting agents. The central object of the theory is the class of variable-order fractional financial networks, whose distinctive feature is that the memory of the market changes over time. The presentation is organized in three stages. First, the modeling assumptions are stated and the governing equations of the network are introduced. Second, the mathematical properties of the model are analyzed: existence and uniqueness of solutions, boundedness of the coupled flow, and stability of the synchronized manifold. Third, a variable-order L^1 scheme is used to compute the Lyapunov spectrum, revealing that the base fractional order controls global network complexity, while the modulation amplitude drives the system between synchronized and desynchronized regimes. As concrete illustrations of the framework, several Bouali-type financial models with investment cycles, policy feedback, and time-varying memory are shown to be special cases. The proposed financial network formulation brings together memory effects of variable fractional order, multi-field coupled agents, and endogenous policy feedback.

Keywords: Financial modeling, Fractional dynamical systems, Variable-order Caputo derivative, Financial networks, Coupled agents, Lyapunov spectrum, Systemic risk

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Direct and exchange channels in relativistic elastic electron scattering from ground-state atomic hydrogen

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Abstract:

In this paper, we investigate laser-assisted elastic scattering of electrons from atomic hydrogen in its ground state in both the nonrelativistic and relativistic regimes, whether the direct channel or the exchange effect channel. This leads to the emergence of two distinct scattering pathways. Hence, this approach requires two separate calculations. The spatial part of the electron scattering wave function is solved numerically while taking electron-exchange interactions into account. These results show that, at low scattering energies $E < 0.07$ (a.u.), the exchange channel dominates. In contrast, for $E \geq 0.07$ (a.u.), the direct channel becomes dominant. It is this different energy behavior which distinguishes the two. On the other hand, Coulomb focusing has a significant influence on incident electrons at low kinetic energies. Furthermore, we already observe a key difference between the approximations involved in nonrelativistic and relativistic calculations at kinetic energy $E > 100$ (a.u.).

Keywords: Hydrogen atom, nonrelativistic and relativistic quantum mechanics, direct and exchange effect channels, calculations DCS

Enhancing Multipartite Entanglement in an OPA-Assisted Rhombus Optomechanical Ring Cavity via Coulomb Coupling

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Abstract:

We investigate the generation and enhancement of bipartite and tripartite entanglement in an optomechanical ring cavity containing an intracavity optical parametric amplifier (OPA). The system consists of three movable charged mirrors coupled to a single cavity mode through geometry-dependent radiation pressure, while the mechanical resonators are also directly coupled through Coulomb interaction. Starting from the total Hamiltonian, we derive the quantum Langevin equations and linearize the dynamics around the steady state. The stationary covariance matrix is then obtained from the Lyapunov equation and used to quantify the quantum correlations through the logarithmic negativity and the minimum residual contangle. Our results show that the cavity-mechanical entanglement is strongly affected by the effective detuning, temperature, OPA gain, pump phase, and Coulomb coupling. The central mechanical resonator exhibits the strongest and most robust entanglement with the cavity field, owing to its larger geometry-dependent radiation-pressure coupling, while the strongest mechanical entanglement is obtained between the two outer mirrors. We also show that the OPA can enhance both bipartite and tripartite correlations when its gain and phase are properly tuned within the stable parameter region. In particular, the hybrid subsystem formed by the central mirror, the third mirror, and the cavity field provides the strongest tripartite entanglement. The limiting cases confirm that the full correlation structure arises from the cooperative action of the phase-sensitive OPA and the asymmetric three-link Coulomb configuration. These results highlight the usefulness of combining parametric amplification, Coulomb coupling, and the rhombus geometry as complementary mechanisms for controlling multipartite quantum correlations in optomechanical ring cavities.

Keywords: Optomechanical ring cavity, optical parametric amplifier, Coulomb coupling, bipartite entanglement, tripartite entanglement, logarithmic negativity, minimum residual contangle

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Asynchronous Exponential Growth for Linear and Nonlinear Functional Differential Equations with Nondense Domains and Finite Delay

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Abstract:

The main objective of this work is to investigate the asynchronous exponential growth (A.E.G.) type dynamics for the linear and nonlinear semigroup generated respectively by an abstract linear and semilinear functional differential equation with a finite delay and a nondense domain. We establish that the nonlinear semigroup inherits the A.E.G. property from its linear counterpart, provided the nonlinear perturbation satisfies appropriate growth and Lipschitz conditions. Specifically, we discuss the asymptotic behavior of solutions for both linear and nonlinear partial functional differential equations (PFDEs) with finite delay. The theoretical results are supported by the analysis of a linear and nonlinear age-structured population model with a delayed birth process, and numerical simulations are provided to illustrate the convergence to a stable age distribution alongside exponential growth in magnitude.

Keywords: Asynchronous exponential growth, partial functional differential equations, finite delay

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Oral Session 9. Probability, Statistics and Stochastic Processes

Statistical Inference for a New Inhomogeneous Gompertz Diffusion Process

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Abstract:

This paper compares homogeneous and inhomogeneous stochastic Gompertz models. First, we review the main characteristics of each model, such as the analytical solution and the transition probability density function. Then, we apply both models to the same real-world data using the Maximum Likelihood Estimation method. The results show that the homogeneous model is simpler and more stable, while the inhomogeneous model is more flexible for time-varying data.

Keywords: Gompertz diffusion process, Statistical inference, Application

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Statistical Inference for Partially Observed Stochastic SIR Epidemics Using the Skellam Distribution

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Abstract:

We consider statistical inference for a stochastic Susceptible–Infectious–Recovered (SIR) epidemic model under partial observation, with particular emphasis on the estimation of the basic reproduction number R_0 . The epidemic dynamics are described by a continuous-time Markov chain in which infection and recovery events are represented through independent Poisson counting processes. Since the net change in the infectious compartment results from the difference between infection and recovery counts, the corresponding transition structure can be characterized using a bivariate Skellam distribution. Based on this representation, we first derive an explicit likelihood for the discretely observed SIR process. In the case of complete observation, a frequentist approach leads to a closed-form maximum likelihood estimator of the basic reproduction number R_0 . The weak consistency and asymptotic normality of this estimator are established under suitable regularity conditions for density-dependent Markov processes. We then address the more realistic situation in which the infectious trajectory is not completely observed. A Bayesian data-augmentation framework is introduced to jointly reconstruct the latent infectious path and estimate R_0 . The resulting posterior distribution is explored using a Markov chain Monte Carlo algorithm combining latent state updates with parameter simulation. This formulation provides a natural quantification of the uncertainty induced by incomplete epidemic observations. The proposed methodology is assessed through stochastic simulations generated by Gillespie's algorithm and is subsequently illustrated using data from the Ebola epidemic in Sierra Leone. The numerical results show that the Skellam-based formulation provides a tractable framework for both likelihood and Bayesian inference in partially observed stochastic epidemic models.

Keywords: Stochastic SIR model, Skellam distribution, Bayesian inference, Maximum likelihood estimation, MCMC, Data augmentation, Basic reproduction number

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Parametric Inference for Linear Drift Models Driven by Tempered Fractional Brownian Motion

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Abstract:

Stochastic processes driven by tempered fractional Brownian motion (tfBm) provide a realistic framework for modeling environments with semi-long-range dependence. Unlike standard fractional Brownian motion, tfBm ensures bounded variance and finite memory via exponential tempering. In this work, we consider the stochastic process $\mathbf{X} = \{X_t, t \geq 0\}$ defined by:

$$X_t = \theta t + \sigma B_{H,\lambda}^I(t),$$

where θ is the linear drift, σ is the volatility, and $(B_{H,\lambda}^I, t \geq 0)$ is a tempered fractional Brownian motion of the first kind governed by the Hurst memory parameter $H \in (0, 1)$ and the exponential tempering parameter $\lambda > 0$.

We develop a joint Maximum Likelihood Estimator (MLE) for the complete parameter vector $\psi = (\theta, \sigma^2, H, \lambda)$ under discrete-time observation. To overcome the severe computational bottlenecks associated with the non-stationarity of tfBm, we utilize the increment process Tempered Fractional Gaussian Noise (TFGN) to guarantee a Toeplitz covariance structure. This transformation enables the application of standard asymptotic theorems for stationary lattice processes, allowing us to formally establish the theoretical consistency and joint asymptotic normality of the estimator.

To validate the theoretical framework, we conduct an extensive Monte Carlo simulation study evaluating the finite-sample performance across diverse stochastic regimes, including anti-persistent memory and high-tempering boundaries. The numerical results confirm the scale-invariance of the proposed numerical approach and demonstrate up to a 33x efficiency gain over standard Ordinary Least Squares (OLS) methods in drift estimation, establishing a highly robust algorithmic foundation for calibrating tfBm models.

Keywords: Tempered Fractional Brownian Motion, Maximum Likelihood Estimation, Fractional Gaussian Noise, Asymptotic Normality, Parameter Estimation

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Copula-Tail Selective Mask Sharpening for Single-Mixture Blind Source Separation: A Multi-Mixture Controlled Study

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Abstract:

Single-mixture blind source separation is underdetermined and therefore depends on structural assumptions. This article evaluates a rank-based upper-tail refinement of blind time-frequency masks. Empirical marginal ranks define copula coordinates, and a smooth joint-tail gate selectively sharpens bins in which several candidate components are simultaneously strong. An initial 15-seed benchmark is complemented by a factorial study with four mixture conditions and six dependence structures: independence, Gaussian, Frank, Clayton, survival Clayton, and Gumbel. Dependent copulas are matched at Kendall's $\tau \approx 0.5$, and all reported combinations use 15 held-out seeds, yielding 360 test cases. In the factorial study, tail-selective sharpening improves scale-invariant signal-to-distortion ratio (SI-SDR) by 4.133 dB over the unrefined soft mask (bootstrap 95% confidence interval: 3.952–4.354 dB). Its advantage over independently calibrated global sharpening is only 0.0159 dB (95% confidence interval: 0.0155–0.0162 dB). Although this small difference is statistically detectable, it is not practically meaningful, and it remains nearly constant across copula families, including independence. Hard masking is 0.695 dB better on average. The results demonstrate the value of sharpening this baseline but do not establish a family-sensitive or scientifically meaningful copula-tail advantage.

Keywords: Blind source separation, Single-channel audio, Copula tail, Time-frequency masking, SI-SDR, Reproducible evaluation

Nonparametric Estimation of the Distribution Function of Random Effects in Fractional Stochastic Differential Equations

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Abstract:

This paper considers the nonparametric estimation of the distribution function of random effects in fractional stochastic differential equations of the form

$$dX_t^i = (a(X_t^i) + \psi_i b(t)) dt + \sigma(t) dB_t^{\mathcal{H},i}, \quad i = 1, \dots, N.$$

with $\mathcal{H}$ is a random hurst in $(0, 1)$. We first estimate the individual random effects (ψ_i) from the observed sample paths and then construct a nonparametric estimator of their distribution function using Bernstein polynomials. We studies the asymptotic properties of the proposed estimator and derive its asymptotic bias and variance. Furthermore, we establish the relationship between the estimated distribution function based on the estimators of the random effects and the true distribution function of the random effects.

Keywords: nonparametric estimation, fractional stochastic differential equations, random effects, Bernstein polynomials, asymptotic bias, asymptotic variance

effect in SDEF using Bernstein polynomial

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Abstract:

This paper considers the nonparametric estimation of the distribution function of random effects in fractional stochastic differential equations of the form

$$dX_t^i = (a(X_t^i) + \psi_i b(t)) dt + \sigma(t) dB_t^{\mathcal{H},i}, \quad i = 1, \dots, N.$$

with $\mathcal{H}$ is a random hurst in $(0, 1)$. We first estimate the individual random effects (ψ_i) from the observed sample paths and then construct a nonparametric estimator of their distribution function using Bernstein polynomials. We studies the asymptotic properties of the proposed estimator and derive its asymptotic bias and variance. Furthermore, we establish the relationship between the estimated distribution function based on the estimators of the random effects and the true distribution function of the random effects.

Nonparametric estimation for fractional Geometric model with random effects, with application to the Nasdaq-100 stock prices

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Abstract:

This paper deals with the decongestion density estimators of the random effects in the drift of geometric fractional Brownian motion. Specifically, we build density estimators based on deconvolution tools; these estimators depend on two tuning parameters, which are selected in a data-driven way, and we study their mean integrated squared error when the Hurst parameter H belongs to $(0, 1)$ and the number of subjects N tends to infinity. Then, we prove the consistency of the obtained estimators not only with long-range dependence but also with the short-range dependence property. The performance of the proposed approach is examined through simulation. Moreover, we apply the proposed approach to analyzing the log return from real Nasdaq 100 index financial data.

Keywords: Random effects model, Fractional Brownian motion, Nonparametric estimation

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Nonparametric estimation for Geometric model with random effects. An application to the stock prices of the Nasdaq-100 index

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Abstract:

This abstract presents the density estimators of the random effects in the drift of geometric fractional Brownian motion. We prove the consistency of the obtained estimators not only with the long-range dependence but also with the short-range dependence property. The performance of the proposed approach is examined through simulation. Moreover, we apply the proposed approach to analyzing the log return from real Nasdaq 100 index financial data.

VAMOS-E: ANYTIME-VALID MULTI-MODEL ORCHESTRATION WITH E-PROCESS CERTIFICATES, DEPENDENCE-AWARE EVIDENCE, AND WEAKLY SUBMODULAR ACQUISITION

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Abstract:

Modern inference systems answer queries by sequentially consulting a pool of heterogeneous, individually unreliable experts—for instance large language model endpoints—whose errors are correlated and whose automatic verifiers are themselves noisy. We formulate the resulting decide-as-you-pay problem as a sequential hypothesis test executed separately on every query: evidence about the current candidate answer accumulates as a nonnegative test martingale, and the system answers only when the associated e -certificate exceeds $1/\alpha_q$. By Ville’s inequality the probability of certifying a wrong answer is at most α_q at any data-dependent stopping time, uniformly over adaptive acquisition policies—a guarantee that fixed-sample calibration methods such as risk-controlling prediction sets cannot provide once stopping is adaptive or the environment drifts. The framework, V amos-E (verifier-aware multi-model orchestration with e -process certificates), replaces the fixed-sample risk gate of our earlier V AMOS-RL system by per-query certificates carried by the acquisition trajectory itself, and has four components with matching theory: (i) tempered likelihood-ratio e -values built from verifier scores and cross-expert agreement, robust to likelihood misspecification with an explicit $e^{\gamma e^{T\bar{\eta}}}$ inflation bound; (ii) dependence-aware agreement evidence in which the wrong-match likelihood is conditioned on the expert-family structure—we prove that independence-assumed evidence inflates certificates exponentially in the number of same-family agreements; (iii) a greedy acquisition rule maximizing a correlation-discounted marginal gain, for which we establish a weak-submodularity approximation guarantee of $(1 - e^{-\gamma})$ with γ lower-bounded via the smallest sparse eigenvalue of the expert error-correlation matrix; and (iv) a system-level Shiryaev–Roberts e -detector over sparsely audited outcomes that preserves time-uniform selective-risk control under distribution shift. A Wald-type identity predicts the expected evidence cost of certification, $E[T] \approx \log(1/\alpha_q)/KL(f_1\|f_0)$, and our simulations reproduce this law across a factor-40 range of verifier informativeness. In a controlled six-expert testbed with family-correlated blind spots and generator–verifier dependence, V amos-E at matched cost attains accuracy 0.835 ± 0.004 with selective risk $0.063 \leq \alpha = 0.10$ and coverage 0.891 , improving simultaneously on accuracy, risk, and coverage over a fixed-sample RCPS-gated sequential router (0.813 ± 0.021 , 0.073 , 0.876); under a mid-stream shift the fixed gate’s cumulative selective risk rises to 0.131 while the monitored system alarms in $8/8$ replicates and ends at $0.090 \leq \alpha$. All numbers are produced by the released reference implementation.

Keywords: e-processes, anytime-valid inference, sequential testing, optional stopping, selective prediction, multi-model orchestration, large language models, verifier calibration, weak submodularity, change detection

Oral Session 10. Networks, Cybersecurity, Cryptography and IoT

Impact of Interaction Graph Construction Topologies on Scalability and Safety in GNN-RL UAV Swarm Path Planning

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Abstract:

The integration of Graph Neural Networks (GNNs) and Multi-Agent Reinforcement Learning (MARL) has significantly advanced autonomous Unmanned Aerial Vehicle (UAV) swarm coordination. However, existing literature primarily focuses on policy architectures or reward shaping, overlooking the critical impact of the underlying interaction graph construction method. This paper presents a systematic comparative study of three core interaction topology paradigms feeding GNN/GAT modules: fixed communication radius (R_{comm}) spatial graphs, K-Nearest Neighbors (K-NN) dynamic graphs, and conflict-driven selective attention-filtered (GAT/IGAT) graphs. We evaluate these approaches across computational complexity, safety margins. Quantitative results demonstrate that fixed-radius graphs exhibit $O(V^2)$ complexity, leading to severe communication overhead in high-density swarms. While the K-NN approach bounds computational complexity to $O(V \cdot K)$, it risks filtering out distant yet safety-critical threats. Conversely, selective graph attention (GAT/IGAT) dynamically prunes uncritical links, operating with up to 43.93% fewer active interaction edges and cutting communication overhead by 30–40%. Concurrently, it reduces Loss of Separation (LoS) duration by 10.52% and collision rates by up to 80%. This study quantifies the pivotal role of initial topology design, providing a practical framework for bandwidth-efficient and secure large-scale UAV swarm deployment.

Keywords: UAV Swarms, Graph neural networks, Graph Attention Networks, K-Nearest Neighbors, Multi-Agent Reinforcement Learning

Communication-Budget-Dependent Trade-offs in Federated IoT Intrusion Detection

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Abstract:

Federated learning (FL) enables intrusion detection systems (IDS) to train across distributed IoT deployments without centralizing raw traffic, but communication cost remains a bottleneck for bandwidth-constrained IoT settings. Fed-ANIDS showed that a FedProx-trained autoencoder (AE) matches centralized IDS performance with no communication compression; we study what happens when compression is applied. Using a FedProx-trained AE on CICIOT2023, a public IoT-traffic intrusion dataset, we apply quantization and top-k sparsification to client updates across ten seeds, two communication budgets (50 and 150 rounds), and IID client partitioning. Aggregate accuracy and F1 are near-invariant to compression, but attack-category-conditional analysis reveals a budget-dependent reversal: sparsification degrades detection of low-volume stealthy attacks at a 50-round budget (miss rate rising to 94.63% at 10% top-k) but improves it at a 150-round budget (miss rate falling to 71.85% vs. 79.14% uncompressed), at the cost of a higher benign false-positive rate. This reversal is consistent across all ten seeds at both budgets. We trace it to the relative inflation rates of the anomaly threshold versus the underlying reconstruction-error distribution, which jointly determine the detector's operating point at a given communication budget, a mechanism that also explains why flood-type attacks (95.97% of the attack test set) stay above 99.99% detection throughout, regardless of compression, and why FedProx's proximal term, compared against plain FedAvg, improves stealthy-attack detection even under IID partitioning.

Keywords: Federated Learning, Intrusion Detection System, IoT Security, Gradient Compression, Autoencoder, Anomaly Detection, FedProx, Communication-Efficient Learning

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Reinforcement Learning-Based Adaptive Cluster Head Selection for Energy-Efficient Wireless Sensor Networks

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Abstract:

Wireless Sensor Networks (WSNs) play a significant role in intelligent infrastructures such as smart cities and Internet of Things systems, yet their performance is strongly limited by energy constraints and unbalanced clustering mechanisms. Classical protocols including LEACH, SEP, and DEEC rely on probabilistic or residual-energy-based Cluster Head (CH) selection, which does not dynamically account for traffic load or transmission delay, leading to energy dissipation and then reduced network lifetime. This paper proposes a Reinforcement Learning (RL)-based adaptive CH selection that jointly optimizes network lifetime, throughput, and transmission delay. The CH selection process is formulated as a Markov Decision Process, where the state space integrates residual node energy, average network energy, node density, and distance to the base station. A centralized Q-learning algorithm executed at the base station determines optimal CH configurations at each round. An analytical lifetime–delay tradeoff model is derived, and a Lyapunov-based energy variance analysis is introduced to demonstrate energy convergence under the learned policy. Simulation results under varying node densities and initial energy levels show more improvements in network lifetime and throughput while maintaining controlled delay compared to classical clustering protocols.

Keywords: Wireless Sensor Networks (WSNs), Deep Reinforcement Learning, Deep Q-Network (DQN), Cluster Head Selection, Energy Efficiency, Network Lifetime, LEACH

Constrained Markov Decision Processes with Learned Lagrange Multipliers for Multi-Agent Task Offloading in Queue-Aware Edge Computing

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Abstract:

Multi-access edge computing (MEC) allows mobile devices to offload computation-intensive and delay-sensitive tasks to nearby edge servers. Jointly optimizing offloading decisions and resource allocation in multi-user, multi-server MEC systems under Quality-of-Service (QoS) constraints leads to a mixed-integer nonlinear program that is NP-hard and must be solved under random channels, task arrivals and server loads. Multi-agent deep reinforcement learning offers a scalable approach, but existing methods usually enforce QoS constraints through a fixed penalty weight in the reward, which requires manual tuning and yields either overly conservative or constraint-violating policies. We formulate the problem as a constrained Markov decision process (CMDP) with per-device latency and energy constraints, and model each MEC server as an M/M/1 queue, so that the waiting time and utilization of each server capture congestion and are included in the agents' observations. We then propose a Constrained Multi-Agent Proximal Policy Optimization (C-MAPPO) algorithm under the centralized training with decentralized execution (CTDE) paradigm [1], which extends our previous single-agent constrained framework to the multi-agent setting. Instead of a fixed penalty, the latency and energy constraints are handled by Lagrange multipliers that are learned during training through projected dual ascent [2], based on the average constraint violation of each agent. The penalty thus increases only when constraints are violated, which removes the need for manual tuning. Simulations with three heterogeneous MEC servers and 3 to 15 mobile devices show that C-MAPPO achieves QoS performance comparable to MAPPO with a hand-tuned penalty weight, while removing the need to tune this weight, and outperforms random offloading and local execution in terms of QoS violation at low and moderate loads. The learned delay multiplier increases progressively during training, whereas the energy multiplier remains close to zero, showing that the mechanism automatically identifies which constraints are binding. Under high load (10 devices), C-MAPPO reduces the average delay violation compared with the fixed-penalty approach, while the results also reveal a server saturation regime in which all methods exceed the deadline, pointing to the need for capacity-aware admission control.

Keywords: Multi-access edge computing, task offloading, constrained Markov decision process, multi-agent reinforcement learning, Lagrangian relaxation, queueing theory

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Twisted Hessian Curve ElGamal Cryptosystem

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Abstract:

In this work, we propose a new public-key encryption scheme based on an ElGamal-type cryptosystem defined over twisted Hessian curves $H_{2a,d}$ over the ring of dual numbers $R = \mathbb{F}_q[X]/(X^2)$, where $q = p^n$ for a prime $p \geq 5$. We first establish the structural properties of $H_{2a,d}(R)$ by formalizing its complete addition formulas and projecting its point group onto $H_{2a,d}(\mathbb{F}_q) \times \mathbb{F}_q$. Utilizing this algebraic isomorphism, we define key generation, encryption, and decryption algorithms that exploit the unified addition laws of twisted Hessian forms. We evaluate the security of the system by reducing the Discrete Logarithm Problem (DLP) on $H_{2a,d}(R)$ to the classical DLP on $H_{2a,d}(\mathbb{F}_q)$ and an additive DLP over $\mathbb{F}_q$. Finally, we show that the complete addition laws eliminate special-case point handling, yielding a 15–20% efficiency improvement in scalar multiplication and enhanced resistance against Simple Power Analysis side-channel attacks compared to standard Weierstrass curves over dual rings.

Keywords: Twisted Hessian curve, Finite ring, Elliptic curve, ElGamal, Cryptography

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Machine Learning and Graph-Based Learning for IoT Threat Detection: A State-of-the-Art Review

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Abstract:

The expansion of the Internet of Things (IoT) has produced highly distributed environments composed of heterogeneous devices, services, and communication technologies, increasing their exposure to evolving cyber threats. Feature-based machine learning (ML) and deep learning (DL) methods detect these threats by learning behavioral and temporal patterns from network traffic, device activities, and system events. Graph-based approaches instead represent entities and their relationships explicitly, allowing structural and relational information to be exploited during detection. Graph Neural Networks (GNNs) combine these perspectives by applying machine learning to graph-structured data, learning from both entity attributes and their neighborhood structure. This paper presents a state-of-the-art review of feature-based ML/DL and graph-based learning approaches for IoT threat detection, analyzing their contributions, capabilities, and limitations with particular attention to the type of information each exploits. The review indicates that no single representation is sufficient across threat types: attacks characterized mainly by anomalous behavior are well addressed by feature-based models, whereas attacks involving coordination, propagation, or unusual communication structures benefit from explicit relational modeling. Hybrid and multi-view approaches are therefore discussed as a direction for integrating complementary behavioral, temporal, and relational information.

Keywords: Internet of Things, Threat detection, Machine learning, Deep learning, Graph analysis, Graph neural networks

Adaptive Threshold Triggering of a Hybrid Genetic–Evolution Strategy for Dynamic Fog-IoT Task Offloading

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Abstract:

Managing resources in IoT networks is more challenging than it might initially appear. Devices frequently run low on battery, tasks accumulate rapidly, and the system must continue operating reliably despite these constraints. While offloading tasks to fog nodes offers a viable solution, most existing offloading methods are built on a critical limitation: they optimize once and assume a static environment. In reality, IoT networks are highly dynamic. To address this gap, we propose an adaptive hybrid strategy that continuously adjusts to system changes. Our approach combines two evolutionary algorithms that alternate based on the degree of environmental shift. Specifically, we employ a Genetic Algorithm (GA) when the system requires a comprehensive re-optimization, and an Evolution Strategy (ES) for minor, incremental adjustments. A thresholdbased mechanism monitors the relative change in the optimization objective between consecutive time steps and automatically selects the appropriate strategy: ES handles small variations, while GA is triggered for significant disruptions. We validate our method through extensive simulations across a wide range of experimental configurations, varying the threshold parameter θ , the population size P , and the number of fog nodes F , on a workload of 500 IoT tasks. The results demonstrate that the hybrid GA–ES strategy consistently reduces the joint energy-delay cost. Compared to pure GA, pure ES, and random allocation baselines, our proposed method outperforms Pure ES by 44.7% and Random Allocation by 13.9%, while requiring 55.7% fewer fitness evaluations than Pure GA, under a realistic dynamic environment with channel variation, random node failures, and scheduled task bursts.

A Privacy-Preserving Dynamic Bridge Model for Multi-Blockchain Interoperability in Supply Chain Traceability

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Abstract:

Multi-blockchain interoperability is essential for end-to-end traceability in heterogeneous supply chain ecosystems. The Dynamic Bridge Model (DBM-SCN) enables this through proof-based ephemeral bridges, yet exhibits a critical limitation as transaction data remain exposed during cross-chain transmission. This exposure creates significant privacy vulnerabilities and violates strict data confidentiality requirements inherent to supply chain operations. This paper proposes ZK-DBM, a Zero-Knowledge Proof-based Dynamic Bridge Model that eliminates data exposure while preserving the “one bridge, one proof, one event” paradigm. The protocol replaces explicit data exchange with cryptographic commitments and zk-SNARK proofs, enabling actors to prove transaction validity without disclosure. We establish four formal security properties (data confidentiality, integrity, chain continuity, and actor sovereignty) while maintaining the six phases of the ephemeral bridge protocol structure. Experimental evaluation on a four-actor chain (100 lots, Ganache-simulated blockchains) demonstrates 2437.74 ms end-to-end latency at 0.7501 TPS, with ZK-Prove (901.14 ms) and ZK-Verify (726.12 ms) achieving practical throughput, while linear gas scaling (43,149 units per transfer) ensures predictable deployment costs. ZK-DBM provides the first unified framework achieving simultaneous dynamic interoperability, strong confidentiality, and semantic preservation for multi-stakeholder supply chains.

Scale-Free Networks Under Targeted Node Attacks Using Diverse Elimination Strategies

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Abstract:

This abstract evaluates the impact of random hub attacks (RHA) and targeted hub attacks (THA) on a Barabási-Albert scale-free network $N=1000$ using the critical injection rate P_c as a congestion threshold indicator. By comparing packet loss and asymmetric packet rejection elimination strategies within a minimal traffic framework, numerical results demonstrate that targeted attacks on as few as two major hubs reduce P_c to zero under the packet loss scheme, causing immediate network congestion. In contrast, adopting a packet rejection mechanism significantly improves network resilience, preserving traffic flow against more than six randomly attacked hubs.

Oral Session 11. Algebra, Functional Analysis and Mathematical Structures

Conformable Fractional Semigroups of Operators with a new infinitesimal generator

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Abstract:

This paper concerns one of the most important tools to solve well-posed problems in the theory of evolution equations (e.g diffusion equation, wave equations, ...) and in the theory of stochastic process, namely the semigroups of linear operators with application to differential equations.

A semigroup of linear operator $\left(T(t)\right)_{t \geq 0}$ on a Banach space X is a continuous operator valued function

$$\begin{cases} T : [0, +\infty) \rightarrow \mathcal{L}(X, X), \\ T(s+t) = T(s)T(t) \text{ for every } t, s \geq 0, \\ T(0) = I. \end{cases} \quad (11.1)$$

The fact that every non zero continuous complex function that satisfies $f(s+t) = f(s)f(t)$ for every $t, s \geq 0$ has the form $f(t) = \exp(at)$, and that f is determined by the number $a = f'(0)$, motivates the association to $\left(T(t)\right)_{t \geq 0}$ of an operator A defined by

$$Ax := \lim_{t \rightarrow 0^+} \frac{T(t)x - x}{t}; \quad x \in \mathcal{D}(A)$$

and called the infinitesimal generator of $\left(T(t)\right)_{t \geq 0}$.

The object of this paper is to introduce a (conformable) fractional semigroup of operators whose generator will be the fractional derivative of the semi-group at $t = 0$. The basic properties of such semigroups will be studied.

Some Characterizations of S -Disjointly Weakly Compact Operators and Applications

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Abstract:

We establish several characterizations of S -disjointly weakly compact operators from a Banach lattice into a Banach space. As a consequence, we obtain characterizations of disjointly weakly compact sets. We also discuss some of the consequences derived from these results.

Keywords: Banach lattice, disjointly weakly compact sets, bounded- E_a operators

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Nonuniform Lyapunov Criteria for Chaotic Dynamics of C_0 -Semigroups

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Abstract:

We present a nonuniform Lyapunov framework for studying chaotic dynamics of C_0 -semigroups on separable Banach spaces. The approach combines forward decay on a dense set with a small-preimage property for dense target states, leading to topological transitivity and hypercyclicity. Quantitative and bounded-gap variants further provide criteria for topological mixing and syndetic transitivity. This formulation emphasizes the complementary roles of asymptotic decay and directional target recovery, while remaining independent of any particular differential equation.

Keywords: C_0 -semigroups, nonuniform Lyapunov criteria, hypercyclicity, topological transitivity, topological mixing, syndetic transitivity

Boundary Values of the Riemann–Liouville Semigroup on Little Favard Spaces

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Abstract:

Let X be a complex Banach space, let $1 \leq p < \infty$, and let $(S(t))_{t \geq 0}$ be the nilpotent right-translation semigroup on $L^p(0, 1; X)$. For $0 < \alpha < 1$, we study the Riemann–Liouville holomorphic semigroup $(J(z))_{\operatorname{Re} z > 0}$ on the little Favard space $\operatorname{fav}_{\alpha, p}^0(S)$. We first establish an explicit regularization estimate showing that J_α maps $L^p(0, 1; X)$ continuously into the Favard space of order α . We prove that the right translation semigroup is not strongly continuous on the corresponding Favard space, whereas its restriction to the little Favard space is a C_0 -semigroup. The principal estimate shows that $(J(z))_{\operatorname{Re} z > 0}$ is uniformly bounded on sufficiently small right half-discs in the little Favard norm. The boundary-value criterion for holomorphic semigroups therefore yields, for every $s \in \mathbb{R}$, the boundary value

$$J(is)f = \lim_{\varepsilon \downarrow 0} J(\varepsilon + is)f, \quad f \in \operatorname{fav}_{\alpha, p}^0(S).$$

The operators $(J(is))_{s \in \mathbb{R}}$ form a strongly continuous group. The proof is direct, includes $p = 1$, and requires no geometric assumption on X .

Keywords: semi-groupe holomorphe, Riemann–Liouville, espaces de Favard, translation à droite, L^p -espaces, Banach complexes, valeurs au bord, C_0 -semi-groupe

Faithful Unit-Distance Realizations of Zero-Divisor Graphs of Finite Products of Fields

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Abstract:

Let $R = F_{q_1} \times \cdots \times F_{q_m}$, $m \geq 2$, be a finite product of fields, and let $\Gamma(R)$ denote its ordinary zero-divisor graph. We study a Euclidean realization problem for this family: when can the vertices of $\Gamma(R)$ be represented by distinct points of the Euclidean plane so that two vertices are adjacent if and only if the Euclidean distance between their representing points is one? We obtain a complete classification. For a product of two fields, a faithful unit-distance realization exists precisely when one of the fields has order 2, or when both fields have order 3. For three field factors, such a realization exists precisely for $R \cong F_2 \times F_2 \times F_2$. No product of four or more finite fields admits a faithful unit-distance realization in the Euclidean plane. The proof combines the multiplicative structure of finite products of fields with elementary rigidity phenomena from unit-distance geometry. In the two-factor case, the zero-divisor graph reduces to a complete bipartite graph, while larger products force Euclidean configurations that cannot occur in the plane. Explicit coordinate realizations are provided for all admissible cases. These results show that both the number and the orders of the field factors impose sharp restrictions on the Euclidean geometry of the associated zero-divisor graph. Thus the zero-product relation in a finite reduced ring gives rise to an exact realization problem connecting commutative algebra, graph theory, and discrete geometry.

Keywords: zero-divisor graph, finite commutative ring, finite field, faithful unit-distance graph, Euclidean realization, discrete geometry